Paper 150M: Coherent Aligned-Patch and Transition-Layer Control for High-Vorticity Navier–Stokes Amplification

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Abstract

Paper 150M begins the ordinary-channel control layer of the 150-series high-vorticity pinching program for the three-dimensional incompressible Navier–Stokes equations. Paper 150L formulated the universal-entry bridge: dangerous amplification must either enter the primary depletion regime or activate a named visibility channel. Paper 150K supplied the unconditional accounting lemmas needed once a channel is visible and estimated. The present paper studies the first two ordinary channels that must be controlled:

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

The aligned-patch channel R^+_{patch} represents dangerous positive stretching carried by coherent support where the vorticity direction remains favorably aligned with the strain field. This is a natural escape route from immediate pinching: if the unit vorticity direction varies slowly, then the directional-gradient cost

$$|\omega|^2|\nabla n|^2$$

may remain small while positive stretching persists. The transition-layer channel R_{trans} represents a related but distinct route: a stretching-active core may remain protected by a boundary, shell, or transition region that delays leakage, deformation, or loss of alignment.

The goal of Paper 150M is not to prove unconditional Navier–Stokes regularity. Its purpose is to formulate control targets for these two coherent channels and to identify the analytic mechanisms by which they may become absorbable. The desired estimates have the form

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t),$$

and

$$R_{\mathrm{trans}}(t) \le \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_\omega(t),$$

or corresponding subinterval-stable integrated estimates. These estimates are useful only if their coefficients preserve the final dissipation margin in the Paper 150J assembly.

The paper develops a classical framework for coherent aligned patches, broad aligned support, protected cores, transition layers, leakage, interface cost, and channel transitions. It also separates visibility from control: activation of R^+_{patch} or R_{trans} does not by itself imply absorbability. Persistent coherence is compared to a characteristic stretching time, since a patch that survives across many stretching times is more dangerous than a short-lived visible patch. Transition-layer leakage is treated as a diagnostic that must be robust under smooth changes of the core-layer mask, especially for diffuse or broad transition regions.

The paper also treats pressure and nonlocal strain explicitly. Coherent alignment may be sustained by nonlocal feedback among separated patches, so the appropriate control object may be a coupled multi-patch support rather than a single local patch. Finally, the coherent-channel coefficient

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

is required to leave dissipation reserve for downstream channels. Thus Paper 150M converts the two most coherent ordinary channels into explicit theorem targets while preserving the modular bridge structure of the 150-series.

1 Introduction

The three-dimensional incompressible Navier–Stokes equations are

$$\partial_t u + (u \cdot \nabla) u + \nabla p = \nu \Delta u, \qquad \nabla \cdot u = 0,$$

posed here on the periodic domain

$$\Omega = \mathbb{T}^3.$$

The vorticity is

$$\omega = \nabla \times u,$$

and satisfies

$$\partial_t \omega + (u \cdot \nabla)\omega = (\omega \cdot \nabla) u + \nu \Delta \omega.$$

The nonlinear term

$$(\omega \cdot \nabla) u$$

is vortex stretching. It is the core three-dimensional amplification mechanism in the Navier–Stokes regularity problem. The formulation used here is classical; see standard references on the Navier–Stokes regularity problem, vorticity methods, and incompressible flow [1, 2, 6, 7].

For smooth solutions, the enstrophy is

$$E_\omega(t) = \frac{1}{2} \int_\Omega |\omega|^2 \, \mathrm{d}V.$$

The classical enstrophy balance is

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}t} = P(t) - D(t),$$

where

$$P(t) = \int_\Omega \omega_i S_{ij} \omega_j \, \mathrm{d}V$$

is vortex stretching and

$$D(t) = \nu \int_\Omega |\nabla \omega|^2 \, \mathrm{d}V$$

is viscous enstrophy dissipation.

The 150-series studies a conditional route toward enstrophy closure based on high-vorticity geometry. Where

$$|\omega| > 0,$$

write

$$\omega = |\omega| n, \qquad n = \frac{\omega}{|\omega|}.$$

The local strain-alignment factor is

$$a(x,t) = n_i S_{ij} n_j,$$

and its positive part is

$$a_+(x,t) = \max\{a(x,t), 0\}.$$

The positive stretching density is

$$|\omega|^2 a_+(x,t).$$

Thus dangerous amplification requires more than large vorticity. It requires large or persistent positive strain alignment that contributes to enstrophy growth.

The geometric cost available in the enstrophy balance is encoded in the identity

$$|\nabla\omega|^2 = |\nabla|\omega||^2 + |\omega|^2|\nabla n|^2.$$

The first term,

$$|\nabla|\omega||^2,$$

is magnitude-gradient cost. The second term,

$$|\omega|^2|\nabla n|^2,$$

is directional-gradient cost. The high-vorticity pinching idea is that dangerous stretching may eventually undermine itself by producing magnitude gradients, directional gradients, leakage, fragmentation, scale-local transfer, or residual pathological concentration.

Papers 150J, 150K, and 150L separated the proof architecture into layers. Paper 150J assembled the conditional closure theorem: if dangerous routes become visible, if their channel estimates are controlled, and if the total dissipation margin remains positive, then enstrophy remains bounded. Paper 150K proved the unconditional accounting layer: partitions, bounded-overlap budgets, pointwise and integrated Gronwall closure, burst summability, threshold splitting, and pathological-reduction bookkeeping. Paper 150L addressed the first deep bridge: dangerous amplification must become visible as primary entry or as one of the named channels,

$$R^+_{\mathrm{patch}}, \quad R_{\mathrm{trans}}, \quad R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

The present paper begins the next bridge layer: ordinary-channel control. It focuses on the first two ordinary channels:

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

These are the most coherent escape routes. The aligned-patch channel represents support on which vorticity direction remains organized and favorably aligned with strain. The transition-layer channel represents a stretching-active core protected by a surrounding layer that delays leakage, deformation, or loss of alignment.

These channels are difficult precisely because they can preserve positive stretching efficiently. If the vorticity direction is coherent, then the directional-gradient cost

$$|\omega|^2|\nabla n|^2$$

may be small. If a coherent core is protected by a transition layer, then stretching may persist before leakage or fragmentation becomes visible. Thus R^+_{patch} and R_{trans} are not nuisances; they are the first serious ordinary-channel obstructions to the final assembly.

The desired control targets are estimates of the form

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t),$$

and

$$R_{\mathrm{trans}}(t) \le \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_\omega(t).$$

For moving, intermittent, or slowly leaking structures, a pointwise estimate may be too strong. In that case, the target is a subinterval-stable integrated estimate:

$$\int_{t_0}^{t} R^+_{\mathrm{patch}}(s)\ \mathrm{d}s \le \delta_{\mathrm{patch}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\mathrm{patch}} \int_{t_0}^{t} E_\omega(s)\ \mathrm{d}s + C^0_{\mathrm{patch}},$$

and similarly for

$$R_{\mathrm{trans}}.$$

Paper 150K explains how such integrated estimates can be used without hiding spikes, provided they hold on all subintervals or on stopping-time partial sums.

The margin requirement remains central. Even if both channels are controlled, the estimates are useful for closure only if the total budget satisfies

$$\theta + \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}} + \delta_{\mathrm{remaining}} < 1.$$

Thus Paper 150M is not merely about bounding R^+_{patch} and R_{trans}. It is about bounding them sharply enough that the final dissipation reserve is not exhausted. In particular, the coherent-channel coefficient

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

must leave a positive reserve for the remaining ordinary and pathological channels. A coherent-channel estimate may succeed locally while still failing the final assembly if it consumes too much of the available dissipation margin.

The paper treats the pressure and nonlocal strain issue directly. The strain tensor

$$S_{ij}$$

is nonlocal through the velocity field and incompressibility. A coherent aligned patch may therefore be sustained by nonlocal strain organization, not only by local vorticity geometry. Paper 150M does not assume away this difficulty. It treats nonlocal strain support as part of the aligned-patch and transition-layer control problem. Any theorem-level estimate must account for how nonlocal strain alignment persists, leaks, fragments, or becomes absorbable.

The paper also allows multi-patch nonlocal feedback. Two or more separated coherent supports may contribute to the strain field experienced by each other. In that case, the correct control object may be a coupled multi-patch support rather than a single local patch. Such a route must either pay joint cost, fragment, become scale-local, enter the complement, or become residual pathology.

The paper also includes broad aligned support. A coherent aligned route need not be a compact intense hotspot. Weak positive alignment over a large region may contribute significantly to the integrated positive-stretching reservoir. If that broad region is coherent, it belongs to the aligned-patch channel. If it is organized through a protected core and surrounding layer, it belongs to the transition-layer channel. If it decomposes into many pieces or scales, later papers on fragmentation and scale-local control become relevant.

A key diagnostic in this paper is persistence. A visible coherent patch is dynamically dangerous only if it persists for a time comparable to the local stretching time. A short-lived patch may be visible but harmless, while a patch that survives across many stretching times can accumulate significant positive stretching. For transition layers, leakage diagnostics must also be robust. A leakage claim should not depend on a jagged or arbitrary core-layer mask. For diffuse transition layers, the relevant diagnostics should be stable under smooth perturbations of the core and transition weights.

The central conditional principle of Paper 150M is:

$$R^+_{\text{patch}} \vee R_{\text{trans}} \quad \Longrightarrow \quad \text{loss of persistence, leakage, fragmentation, scale transfer, or absorbable cost.}$$

In estimate form, the target is:

$$R^+_{\text{patch}} + R_{\text{trans}} \le \delta_{\text{coh}} D + C_{\text{coh}} E_\omega,$$

with

$$\delta_{\text{coh}} = \delta_{\text{patch}} + \delta_{\text{trans}}$$

small enough to preserve the Paper 150J margin after the remaining channels are added.

Finally, the transition-layer terminology is purely fluid-mechanical. A transition layer is not a horizon or a separate causal boundary. It is a region in which gradients, leakage, deformation, or alignment loss mediate the interaction between a stretching-active core and the surrounding flow. Its relevance is analytic: if the layer delays depletion, then it must eventually pay cost, leak, fragment, become scale-local, enter the complement, or become residual pathology.

The paper is organized as follows. Section 2 fixes the classical Navier–Stokes notation and the channel targets. Section 3 defines coherent aligned-patch geometry. Section 4 formulates aligned-patch control targets. Section 5 defines protected cores and transition layers. Section 6 formulates transition-layer control targets. Section 7 studies how patch control can fail into transition layers, fragmentation, scale-local transfer, or pathology. Section 8 states the conditional ordinary-channel control theorem for R^+_{patch} and R_{trans}. Section 9 gives falsifiers and failure modes. Section 10 explains the relation to Papers 150J, 150K, and 150L. Section 11 summarizes the bridge target.

2 Classical Setup and Channel Targets

This section fixes the notation and channel targets for Paper 150M. The paper remains entirely within the classical three-dimensional incompressible Navier–Stokes framework. Its purpose is to study the first two ordinary visibility channels identified by Paper 150L:

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

These channels represent coherent aligned support and protected transition-layer support. They are visibility classes first, and control targets second.

2.1 Navier–Stokes and vorticity form

Let

$$u : \Omega \times [0,T] \to \mathbb{R}^3$$

be a smooth divergence-free velocity field on the periodic domain

$$\Omega = \mathbb{T}^3.$$

The incompressible Navier–Stokes equations are

$$\partial_t u + (u \cdot \nabla)u + \nabla p = \nu \Delta u, \qquad \nabla \cdot u = 0,$$

with kinematic viscosity

$$\nu > 0.$$

The vorticity is

$$\omega = \nabla \times u.$$

Taking the curl gives

$$\partial_t \omega + (u \cdot \nabla)\omega = (\omega \cdot \nabla)u + \nu \Delta \omega.$$

The term

$$(\omega \cdot \nabla)u$$

is vortex stretching.

The strain tensor is

$$S_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i).$$

The total stretching production is

$$P(t) = \int_\Omega \omega_i S_{ij} \omega_j \, \mathrm{d}V.$$

2.2 Enstrophy balance

The enstrophy is

$$E_\omega(t) = \frac{1}{2} \int_\Omega |\omega(x,t)|^2 \, \mathrm{d}V.$$

The viscous enstrophy dissipation is

$$D(t) = \nu \int_\Omega |\nabla \omega(x,t)|^2 \, \mathrm{d}V.$$

For smooth periodic solutions,

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}t} = P(t) - D(t).$$

This balance is the only closure mechanism used in this paper. The goal is to identify conditions under which the coherent-channel contributions can be bounded by dissipation and lower-order enstrophy without exhausting the margin needed by Paper 150J.

2.3 Positive stretching density

Where

$$|\omega| > 0,$$

write

$$\omega = |\omega| n, \qquad n = \frac{\omega}{|\omega|}.$$

The local strain-alignment factor is

$$a(x,t) = n_i S_{ij} n_j.$$

Its positive part is

$$a_+(x,t) = \max\{a(x,t), 0\}.$$

The positive stretching density is

$$|\omega|^2 a_+(x,t).$$

The positive stretching reservoir is

$$P^+(t) = \int_\Omega |\omega|^2 a_+(x,t)\ \mathrm{d}V.$$

The aligned-patch and transition-layer channels are both mechanisms by which this reservoir may remain significant outside immediate primary depletion.

2.4 Gradient-cost decomposition

The enstrophy-gradient density decomposes as

$$|\nabla\omega|^2 = |\nabla|\omega||^2 + |\omega|^2 |\nabla n|^2.$$

Thus

$$D(t) = \nu \int_\Omega |\nabla|\omega||^2\ \mathrm{d}V + \nu \int_\Omega |\omega|^2 |\nabla n|^2\ \mathrm{d}V.$$

The first term is magnitude-gradient cost. The second term is directional-gradient cost. The directional-gradient term is central for aligned-patch control. A coherent aligned patch may preserve positive stretching precisely because

$$|\nabla n|$$

is small on the active support. This makes R^+_{patch} a serious channel rather than an immediately depleted one.

Transition layers enter when a coherent core remains active while the surrounding boundary region begins to carry magnitude-gradient cost, directional-gradient cost, leakage, deformation, or alignment loss.

2.5 Primary estimate and channel remainder

The primary depletion architecture has schematic form

$$P(t) \leq \theta D(t) + CE_\omega(t) + R_\kappa(t), \qquad 0 \leq \theta < 1.$$

The remainder is decomposed as

$$R_\kappa = R^+_{\text{patch}} + R_{\text{trans}} + R_{\text{frag}} + R_{\text{scale}} + R_{\text{low}} + R_{\text{path}}.$$

Paper 150M focuses only on

$$R^+_{\text{patch}} \qquad \text{and} \qquad R_{\text{trans}}.$$

The remaining channels

$$R_{\text{frag}}, \quad R_{\text{scale}}, \quad R_{\text{low}}, \quad R_{\text{path}}$$

are not controlled here except as exit routes or failure modes for the two coherent channels.

2.6 Aligned-patch channel

The aligned-patch channel

$$R^+_{\text{patch}}$$

is activated when dangerous positive stretching is carried by coherent support on which the vorticity direction remains favorably aligned with strain.

A schematic aligned-patch contribution is

$$R^+_{\text{patch}}(t) \sim \int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V,$$

where

$$\Omega_{\text{patch}}(t)$$

is a stretching-active support with coherent vorticity direction and positive strain alignment.

The aligned-patch channel may include compact intense support or broad coherent support. A route does not need to be localized in a small region to be aligned-patch-like. If weak positive alignment is coherent over a large region and the integrated positive stretching is significant, then the broad support may still contribute to

$$R^+_{\text{patch}}.$$

The control target is

$$R^+_{\text{patch}}(t) \leq \delta_{\text{patch}} D(t) + C_{\text{patch}} E_\omega(t),$$

or a subinterval-stable integrated version.

2.7 Transition-layer channel

The transition-layer channel

$$R_{\text{trans}}$$

is activated when a stretching-active core is protected, insulated, or delayed by a surrounding boundary or transition region.

A schematic geometry is

$$\Omega_{\text{core}}(t) \subset \Omega_{\text{core}}(t) \cup \Omega_{\text{trans}}(t),$$

where

$$\Omega_{\text{core}}(t)$$

carries coherent positive stretching and

$$\Omega_{\text{trans}}(t)$$

is a surrounding region where leakage, deformation, interface cost, or alignment loss begins to appear.

A schematic transition-layer contribution is

$$R_{\text{trans}}(t) \sim \int_{\Omega_{\text{core}}(t)\cup\Omega_{\text{trans}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V$$

for the portion not already assigned to the aligned-patch estimate.

The control target is

$$R_{\text{trans}}(t) \leq \delta_{\text{trans}} D(t) + C_{\text{trans}} E_\omega(t),$$

or a subinterval-stable integrated version.

2.8 Combined coherent-channel target

The combined coherent-channel contribution is

$$R_{\text{coh}}(t) = R^+_{\text{patch}}(t) + R_{\text{trans}}(t).$$

The ideal pointwise target is

$$R_{\text{coh}}(t) \leq \delta_{\text{coh}} D(t) + C_{\text{coh}} E_\omega(t),$$

where

$$\delta_{\text{coh}} = \delta_{\text{patch}} + \delta_{\text{trans}}$$

and

$$C_{\text{coh}} = C_{\text{patch}} + C_{\text{trans}}.$$

For moving, intermittent, or slowly leaking structures, the integrated target may be more realistic:

$$\int_{t_0}^t R_{\text{coh}}(s)\ \mathrm{d}s \leq \delta_{\text{coh}} \int_{t_0}^t D(s)\ \mathrm{d}s + C_{\text{coh}} \int_{t_0}^t E_\omega(s)\ \mathrm{d}s + C^0_{\text{coh}}$$

for every

$$t \in I.$$

This subinterval-stability requirement is inherited from Paper 150K. It prevents an integrated estimate from hiding a pointwise spike.

2.9 Margin requirement

The estimates in this paper are useful for the final assembly only if they preserve the dissipation margin. If

$$\theta$$

is the primary depletion coefficient, then the coherent-channel part consumes

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}.$$

The remaining channels consume

$$\delta_{\mathrm{remaining}} = \delta_{\mathrm{frag}} + \delta_{\mathrm{scale}} + \delta_{\mathrm{low}} + \delta_{\mathrm{path}}.$$

The final Paper 150J margin requires

$$\theta + \delta_{\mathrm{coh}} + \delta_{\mathrm{remaining}} < 1.$$

Thus a bound on R^{+}_{patch} or R_{trans} is not enough by itself. The coefficient must be sharp enough to leave room for the remaining channels.

2.10 Preliminary coherent-channel budget discipline

The coherent-channel coefficient

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

cannot be treated as an unlimited reserve. The final Paper 150J assembly requires

$$\theta + \delta_{\mathrm{coh}} + \delta_{\mathrm{remaining}} < 1.$$

Therefore, coherent-channel control is useful only if

$$\delta_{\mathrm{coh}}$$

is substantially below the remaining dissipation budget after primary depletion.

At this stage, Paper 150M does not claim a sharp numerical value for

$$\delta_{\mathrm{coh}}.$$

However, the intended target is not merely

$$\delta_{\mathrm{coh}} < 1 - \theta.$$

The stronger practical target is that coherent-channel control leaves room for the downstream ordinary and pathological channels:

$$\delta_{\mathrm{coh}} \ll 1 - \theta,$$

or at least

$$\delta_{\text{coh}} \leq (1-\theta) - \delta_{\text{reserve}},$$

for a declared positive reserve

$$\delta_{\text{reserve}} > 0$$

available to fragmentation, scale-local, complement, and pathological channels.

This budget discipline prevents the coherent-channel estimates from winning locally while exhausting the global proof margin. Sharp coefficient recovery remains a later bridge problem, but Paper 150M records the necessary constraint.

2.11 Pressure and nonlocal strain

The strain tensor

$$S_{ij}$$

is nonlocal through the velocity field and incompressibility. Therefore, an aligned patch or transition-layer structure may be sustained by nonlocal strain organization rather than purely local vorticity geometry.

This is not treated as a loophole. In Paper 150M, nonlocal strain support belongs directly to the aligned-patch and transition-layer control problem. If nonlocal strain preserves coherent positive stretching on a support, that support contributes to

$$R^{+}_{\text{patch}}$$

or

$$R_{\text{trans}}$$

depending on whether a protected core or transition layer is present.

A theorem-level estimate must therefore account for how nonlocal strain alignment is maintained, how long it can persist, whether it leaks into a transition layer, whether it fragments, or whether it pays magnitude-gradient or directional-gradient cost.

The support may also be nonlocal in a coupled sense. Several separated patches may sustain one another through the strain field. In that case, the appropriate coherent-channel object may be a coupled multi-patch support rather than a single local support. Such a route must either pay joint cost, fragment, become scale-local, enter the complement, or become residual pathology.

2.12 Visibility versus control

Paper 150L established visibility:

$$\mathcal{D}_{\text{amp}}(I) \Longrightarrow \mathcal{E}_{\text{entry}}(I) \vee \mathcal{C}_{\text{chan}}(I).$$

Paper 150M begins control. The distinction is essential.

Visibility means

$$R^{+}_{\text{patch}}(I) \quad \text{or} \quad R_{\text{trans}}(I)$$

has activated. Control means proving estimates such as

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t),$$

or

$$R_{\mathrm{trans}}(t) \le \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_\omega(t).$$

A channel can be visible and still uncontrolled. Paper 150M studies what additional structure would make these coherent channels absorbable.

2.13 What this paper proves and does not prove

Paper 150M formulates control targets for the first two ordinary channels:

$$R^+_{\mathrm{patch}} \quad \text{and} \quad R_{\mathrm{trans}}.$$

It seeks conditional estimates and failure modes for coherent aligned-patch support and protected transition-layer support.

It does not prove full Navier–Stokes regularity. It does not control fragmentation, scale-local transfer, low-vorticity complement stretching, or pathological concentration except as possible exit routes from coherent-channel control. It does not recover the final margin by itself.

The role of Paper 150M is narrower:

$$\text{visible coherent channels} \quad \Longrightarrow \quad \text{control target, exit route, or named obstruction.}$$

2.14 Summary

This section fixed the classical notation, the enstrophy balance, the positive-stretching density, the gradient-cost decomposition, and the two channel targets:

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

The desired outcome is a pointwise or subinterval-stable integrated estimate for

$$R^+_{\mathrm{patch}} + R_{\mathrm{trans}}$$

with coefficients small enough to preserve the Paper 150J dissipation margin. The section also recorded a preliminary budget discipline: coherent-channel control must leave reserve for downstream channels, and nonlocal strain may require coupled multi-patch estimates.

The next section defines coherent aligned-patch geometry.

3 Coherent Aligned-Patch Geometry

The previous section fixed the channel targets for Paper 150M. This section defines the geometric structure of the aligned-patch channel

$$R^+_{\mathrm{patch}}.$$

The aligned-patch channel is activated when dangerous positive stretching is carried by support on which the vorticity direction remains coherent and favorably aligned with strain.

The central difficulty is that coherent alignment can preserve stretching while delaying directional-gradient cost. This makes aligned patches a serious ordinary-channel obstruction rather than an immediately depleted structure.

3.1 Positive stretching on a patch

Let

$$\Omega_{\mathrm{patch}}(t) \subset \Omega$$

be a stretching-active support. The positive stretching carried by this support is

$$P^{+}_{\mathrm{patch}}(t) = \int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V,$$

where

$$a_+(x,t) = \max\{n_i S_{ij} n_j, 0\}.$$

The patch is relevant only if

$$P^{+}_{\mathrm{patch}}(t)$$

is non-negligible relative to the positive-stretching reservoir or to the channel budget under study. A region with high vorticity but negligible positive alignment is not an aligned-patch obstruction.

Thus the aligned-patch channel is not defined by vorticity magnitude alone. It is defined by positive stretching supported on a coherent region.

3.2 Directional coherence

A patch is directionally coherent if the vorticity direction

$$n = \frac{\omega}{|\omega|}$$

varies slowly on the patch. The natural directional-gradient cost is

$$\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 |\nabla n|^2\ \mathrm{d}V.$$

A useful coherence ratio is

$$\mathcal{Q}_{\mathrm{patch}}(t) = \frac{\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 |\nabla n|^2\ \mathrm{d}V}{\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V + \varepsilon}.$$

Small

$$\mathcal{Q}_{\mathrm{patch}}(t)$$

means the patch preserves positive stretching while paying little directional-gradient cost. This is the difficult aligned-patch regime.

Large

$$\mathcal{Q}_{\mathrm{patch}}(t)$$

means the patch is already paying directional cost. In that case, the contribution may be absorbable into dissipation.

3.3 Strain alignment

Directional coherence alone is not enough. The vorticity direction must also align favorably with strain:

$$a(x,t) = n_i S_{ij} n_j.$$

The positive part is

$$a_+(x,t) = \max\{a(x,t), 0\}.$$

A patch is positively aligned if a significant fraction of its weighted vorticity lies where

$$a_+(x,t) > 0.$$

One may define an aligned positive-stretching fraction

$$\Pi^+_{\text{patch}}(t) = \frac{\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |a(x,t)|\ \mathrm{d}V + \varepsilon}.$$

If

$$\Pi^+_{\text{patch}}(t)$$

is close to one, then the patch is dominantly stretching-active rather than cancellation-dominated.

The aligned-patch obstruction is strongest when both conditions hold:

$$\mathcal{Q}_{\text{patch}}(t) \ll 1, \qquad \Pi^+_{\text{patch}}(t) \approx 1.$$

Then the patch carries positive stretching while paying little directional-gradient cost.

3.4 Patch persistence

A coherent patch is dangerous only if it persists long enough to contribute meaningfully to enstrophy growth. Let

$$I = [t_0, t_1]$$

be a smooth interval. A patch family

$$\Omega_{\text{patch}}(t)$$

is persistent on I if the time-integrated positive stretching

$$\int_I P^+_{\text{patch}}(t)\ \mathrm{d}t$$

is significant and if the patch remains trackable by a spatial, material, weighted, or stopping-time diagnostic.

Persistence does not require that the same fixed spatial set remain active for all t. A coherent patch may move. What matters is whether the stretching-active structure can be followed in a meaningful way.

If the patch loses persistence, then the route may become harmless or may exit to another channel:

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad \text{or} \quad R_{\mathrm{path}}.$$

3.5 Persistence time and stretching time

A coherent aligned patch is dangerous only if it persists long enough for positive stretching to accumulate. The natural local comparison time is the stretching time

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty(\Omega_{\mathrm{patch}}(t))} + \varepsilon},$$

or, more locally,

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty(\Omega_{\mathrm{patch}}(t))} + \varepsilon}.$$

If the patch lifetime

$$\tau_{\mathrm{patch}}$$

is much shorter than this stretching time, then the patch may be visible but not dynamically dangerous. If instead

$$\tau_{\mathrm{patch}} \gtrsim \tau_S$$

over a dangerous interval, then coherent alignment has enough time to contribute to enstrophy growth.

Thus aligned-patch control may be approached through a lifetime estimate:

$$\tau_{\mathrm{patch}} \le C_{\mathrm{life}} \tau_S$$

unless the patch pays directional-gradient cost, loses alignment, leaks into a transition layer, fragments, becomes scale-local, enters the complement, or becomes residual pathology.

This is only a heuristic timescale criterion in the present paper. A theorem-level version would require a material or weighted tracking lemma showing that coherent alignment cannot persist over many stretching times without paying one of the named costs.

3.6 Broad aligned support

An aligned patch need not be a small intense hotspot. A broad region with weak but positive alignment may carry significant integrated positive stretching if its measure is large. Such a route was identified in Paper 150L as wide-area low-intensity danger.

Let

$$\Omega_{\mathrm{patch}}(t)$$

have large measure and modest pointwise alignment:

$$0 < a_+(x,t) \ll 1$$

on much of the support. The contribution

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V$$

may still be significant.

If the vorticity direction and strain alignment are coherent over the broad support, then the route belongs to

$$R^+_{\text{patch}}.$$

If the broad support decomposes into many weakly active pieces, then it may instead activate

$$R_{\text{frag}}$$

or

$$R_{\text{scale}}.$$

If the support is near or below a high-vorticity threshold, it may activate

$$R_{\text{low}}.$$

If no ordinary classification applies, it becomes residual pathology:

$$R_{\text{path}}.$$

Thus broad aligned support is included in the aligned-patch geometry, but only when coherence is genuinely present.

3.7 Patch boundaries and leakage

A patch has a boundary or transition region where vorticity magnitude, vorticity direction, or strain alignment changes. If this boundary becomes important, the route may begin to activate the transition-layer channel

$$R_{\text{trans}}.$$

Let

$$\partial\Omega_{\text{patch}}(t)$$

denote the effective boundary of the patch, interpreted in a smooth, weighted, or coarse-grained sense. A schematic boundary cost is

$$\mathcal{B}_{\text{patch}}(t) = \nu \int_{\mathcal{N}_r(\partial\Omega_{\text{patch}}(t))} |\nabla\omega|^2\ \mathrm{d}V,$$

where

$$\mathcal{N}_r(\partial\Omega_{\text{patch}}(t))$$

is a radius-r neighborhood of the boundary.

If the patch preserves stretching while the boundary cost is small, then the aligned-patch obstruction remains strong. If boundary cost grows, the contribution may become absorbable or may transition into

$$R_{\text{trans}}.$$

Boundary diagnostics should be interpreted robustly. For broad or diffuse patches, a sharp boundary may be artificial. In that case, the patch boundary should be replaced by a smooth weight, a coarse-grained interface region, or a stopping-time diagnostic that is stable under small perturbations of the patch mask.

3.8 Material or moving patches

A coherent patch may move through the domain. Fixed spatial masks may fail to track it. For this reason, an aligned patch should be allowed to be material, moving, or weighted.

A moving patch family

$$\Omega_{\text{patch}}(t)$$

is still an aligned patch if:

- it carries significant positive stretching;
- its vorticity direction remains coherent in the moving support;
- strain alignment remains favorable;
- the support remains trackable by a material, weighted, or stopping-time diagnostic.

If moving support cannot be tracked but its cumulative positive stretching remains significant, then the route may exit to

$$R_{\text{path}}.$$

Thus motion is not a loophole. It is either part of aligned-patch geometry or a route into residual pathology.

3.9 Patch visibility versus patch control

Visibility of an aligned patch means

$$R^+_{\text{patch}}(I)$$

is active. It does not mean

$$R^+_{\text{patch}}(t) \leq \delta_{\text{patch}} D(t) + C_{\text{patch}} E_\omega(t).$$

This distinction is central. Paper 150L supplied visibility. Paper 150M studies control targets. The fact that a patch is coherent and aligned may make it visible precisely because it is hard to control.

The control problem is to show that a persistent aligned patch must eventually do at least one of the following:

- pay directional-gradient cost;
- pay magnitude-gradient or boundary cost;

- lose positive strain alignment;
- leak into a transition layer;
- fragment into many components;
- become scale-local;
- move into a threshold/complement channel;
- become residual pathology.

3.10 Definition of coherent aligned patch

We now give the working definition.

Definition 1 (Coherent aligned patch)**.** Let u be a smooth solution on

$$I = [t_0, t_1].$$

A family of supports

$$\Omega_{\text{patch}}(t) \subset \Omega$$

is a coherent aligned patch on I if:

1. the patch carries significant positive stretching:

$$\int_I \int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+(x,t) \ \mathrm{d}V \ \mathrm{d}t$$

   is non-negligible relative to the channel budget;

2. the vorticity direction is coherent on the patch, measured by small or controlled directional-gradient cost:

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2 \ \mathrm{d}V;$$

3. positive strain alignment is significant:

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+(x,t) \ \mathrm{d}V$$

   is not negligible;

4. the support is fixed, moving, material, weighted, or otherwise trackable over the interval or over a stopping-time subinterval;
5. the support persists long enough to be dynamically relevant, measured relative to a local stretching time, or else exits to a harmless or named channel state;
6. the contribution is not already assigned to primary entry, transition-layer support, fragmentation, scale-local transfer, low-vorticity complement stretching, or residual pathology.

This definition is deliberately flexible because coherent patches may be localized, broad, moving, or weighted. The key features are positive stretching, coherent direction, favorable strain alignment, trackable support, and persistence relative to the stretching time.

3.11 Aligned-patch channel contribution

The aligned-patch channel contribution is the part of the remainder assigned to coherent aligned patches:

$$R^+_{\mathrm{patch}}(t).$$

In a partition-of-unity formulation, one may write schematically

$$R^+_{\mathrm{patch}}(t) = \int_\Omega \chi_{\mathrm{patch}}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V,$$

where

$$0 \le \chi_{\mathrm{patch}} \le 1$$

is a measurable channel weight supported on aligned-patch visibility.

The exact form of

$$\chi_{\mathrm{patch}}$$

depends on the chosen diagnostic. Paper 150K supplies the accounting framework: if channel weights overlap, the overlap must be charged explicitly.

3.12 Summary

A coherent aligned patch is a support that carries significant positive stretching while maintaining coherent vorticity direction and favorable strain alignment. Such patches may be localized, broad, moving, or weighted.

The aligned-patch channel is dangerous because it may preserve stretching while paying little directional-gradient cost. This danger depends not only on instantaneous coherence but also on persistence relative to the local stretching time. The next section formulates the control targets for this channel.

4 Aligned-Patch Control Targets

The previous section defined coherent aligned-patch geometry. This section formulates the control targets for the aligned-patch channel

$$R^+_{\mathrm{patch}}.$$

The goal is not to prove unconditional control in this paper. The goal is to state clearly what must be shown for an aligned patch to become absorbable in the Paper 150J assembly.

The aligned-patch channel is difficult because coherent vorticity direction can preserve positive stretching while delaying directional-gradient cost. Therefore, the control target must identify how persistent aligned support eventually pays cost, loses alignment, leaks into a transition layer, fragments, becomes scale-local, moves into the complement, or remains as a named obstruction.

4.1 The aligned-patch obstruction

An aligned patch carries positive stretching of the form

$$R^+_{\text{patch}}(t) \sim \int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V.$$

It becomes an obstruction when this contribution remains significant while the directional-gradient cost

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2\ \mathrm{d}V$$

is small relative to the stretching carried by the patch.

The difficult regime is therefore:

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+\ \mathrm{d}V \quad \text{large},$$

while

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2\ \mathrm{d}V \quad \text{small}.$$

This regime cannot be dismissed by the primary pinching mechanism alone. It is exactly why

$$R^+_{\text{patch}}$$

is separated as an ordinary channel.

4.2 Persistence time and stretching time

A coherent aligned patch is dangerous only if it persists long enough for positive stretching to accumulate. The natural local comparison time is the stretching time

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty(\Omega_{\text{patch}}(t))} + \varepsilon},$$

or, more locally,

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty(\Omega_{\text{patch}}(t))} + \varepsilon}.$$

If the patch lifetime

$$\tau_{\text{patch}}$$

is much shorter than this stretching time, then the patch may be visible but not dynamically dangerous. If instead

$$\tau_{\text{patch}} \gtrsim \tau_S$$

over a dangerous interval, then coherent alignment has enough time to contribute to enstrophy growth.

Thus aligned-patch control may be approached through a lifetime estimate:

$$\tau_{\text{patch}} \le C_{\text{life}} \tau_S$$

unless the patch pays directional-gradient cost, loses alignment, leaks into a transition layer, fragments, becomes scale-local, enters the complement, or becomes residual pathology.

This is only a heuristic timescale criterion in the present paper. A theorem-level version would require a material or weighted tracking lemma showing that coherent alignment cannot persist over many stretching times without paying one of the named costs.

4.3 Pointwise aligned-patch target

The strongest useful target is a pointwise estimate:

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t).$$

Here

$$\delta_{\mathrm{patch}} \ge 0$$

is the dissipation fraction consumed by the aligned-patch channel, and

$$C_{\mathrm{patch}} \ge 0$$

is a lower-order enstrophy coefficient.

This estimate is useful for closure only if the final margin remains positive:

$$\theta + \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}} + \delta_{\mathrm{remaining}} < 1.$$

Thus the estimate must be not only true, but sharp enough to leave room for the transition-layer channel and the remaining channels:

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

4.4 Integrated aligned-patch target

A pointwise estimate may be too strong for moving or intermittent aligned patches. A patch may preserve stretching over a sequence of subintervals, or it may move in a way that makes fixed-time diagnostics unstable. In that case, the relevant target is a subinterval-stable integrated estimate:

$$\int_{t_0}^{t} R^+_{\mathrm{patch}}(s)\ \mathrm{d}s \le \delta_{\mathrm{patch}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\mathrm{patch}} \int_{t_0}^{t} E_\omega(s)\ \mathrm{d}s + C^0_{\mathrm{patch}}$$

for every

$$t \in I.$$

The requirement that this estimate hold for every

$$[t_0, t] \subset I$$

is essential. A full-interval estimate alone may hide an intermediate spike. Paper 150K showed that integrated estimates support closure only when they are subinterval-stable or when a stopping-time partial-sum structure controls all intermediate times.

4.5 Directional-cost route

The most direct control mechanism is directional-gradient cost. If coherent alignment begins to fail, then

$$|\nabla n|$$

increases on or near the patch. The patch contribution may then be controlled by the directional part of dissipation:

$$\nu \int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2 \, \mathrm{d}V.$$

A schematic directional-cost target is:

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+ \, \mathrm{d}V \le c_{\text{dir}} \nu \int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2 \, \mathrm{d}V + C_{\text{dir}} E_\omega(t).$$

If such an estimate holds with a small enough effective coefficient, then

$$R^+_{\text{patch}}$$

is absorbable.

The difficult case is when the patch remains coherent:

$$|\nabla n|$$

stays small while positive stretching remains large. This is the coherent-aligned obstruction.

4.6 Magnitude-gradient and boundary-cost route

Even if the vorticity direction is coherent inside the patch, the patch may have boundaries where vorticity magnitude changes. Magnitude-gradient cost is measured by

$$|\nabla |\omega||^2.$$

A schematic boundary or interface cost is

$$\mathcal{B}_{\text{patch}}(t) = \nu \int_{\mathcal{N}_r(\partial \Omega_{\text{patch}}(t))} |\nabla \omega|^2 \, \mathrm{d}V.$$

If the patch maintains high positive stretching while remaining separated from surrounding flow, then either the boundary must remain smooth and stable, or it must pay interface cost. A possible control route is:

$$R^+_{\text{patch}}(t) \le c_{\text{bd}} \mathcal{B}_{\text{patch}}(t) + C_{\text{bd}} E_\omega(t).$$

If the boundary cost grows, the patch contribution may become absorbable. If the boundary remains protective rather than dissipative, the route may exit to the transition-layer channel:

$$R_{\text{trans}}.$$

For broad or diffuse patches, the boundary cost should not depend on a jagged or arbitrary mask. A robust boundary-cost estimate should survive smooth perturbations of the patch support or be formulated with smooth weights rather than a hard interface.

4.7 Alignment-loss route

A patch may stop being dangerous if positive strain alignment weakens. Recall

$$a(x,t) = n_i S_{ij} n_j, \qquad a_+(x,t) = \max\{a(x,t), 0\}.$$

If the aligned fraction decreases, then

$$\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V$$

becomes smaller even if vorticity magnitude remains large.

A schematic alignment-loss criterion is:

$$\Pi^+_{\mathrm{patch}}(t) = \frac{\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 |a(x,t)|\ \mathrm{d}V + \varepsilon}$$

decreases below a declared threshold. In that case, the patch no longer carries dominantly positive stretching and may cease to be a dangerous aligned-patch contribution.

This route does not control stretching by dissipation directly. It reduces the positive-stretching reservoir itself.

4.8 Nonlocal and multi-patch feedback

The strain tensor

$$S_{ij}$$

is nonlocal. A coherent aligned patch may be sustained by pressure-mediated strain produced partly by vorticity outside the patch. In particular, two or more separated coherent supports may contribute to each other's strain alignment.

A schematic coupled support is

$$\Omega^{\mathrm{coup}}_{\mathrm{patch}}(t) = \bigcup_{m=1}^{M} \Omega^{(m)}_{\mathrm{patch}}(t).$$

The corresponding coupled aligned-patch contribution is

$$R^{+}_{\mathrm{patch}}{}^{\mathrm{coup}}(t) = \int_{\Omega^{\mathrm{coup}}_{\mathrm{patch}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V.$$

In this situation, it may be too strong to require each individual component

$$\Omega^{(m)}_{\mathrm{patch}}(t)$$

to be absorbable on its own. The appropriate theorem target may instead be a coupled estimate:

$$R_{\text{patch}}^{+\ \text{coup}}(t) \le \delta_{\text{coup}} D(t) + C_{\text{coup}} E_\omega(t),$$

or else a named exit to transition-layer, fragmentation, scale-local, complement, or pathological channels.

This prevents nonlocal strain support from being hidden by an overly local definition of patch control. A nonlocal feedback loop is not a loophole, but it may require a joint multi-patch estimate rather than a single-patch estimate.

4.9 Leakage into transition-layer route

A coherent aligned patch may fail by developing a protected boundary or transition region. The patch does not immediately become harmless. Instead, it changes classification:

$$R_{\text{patch}}^{+} \quad \Longrightarrow \quad R_{\text{trans}}.$$

This occurs when positive stretching remains concentrated in a core while leakage, deformation, or alignment loss begins in a surrounding layer:

$$\Omega_{\text{core}}(t) \subset \Omega_{\text{core}}(t) \cup \Omega_{\text{trans}}(t).$$

The route then belongs to the transition-layer channel. The control problem shifts from pure patch coherence to core lifetime, boundary cost, leakage rate, and transition-layer dissipation.

Thus leakage into a transition layer is not failure of the taxonomy. It is a channel transition.

4.10 Fragmentation exit route

A coherent patch may also fail by breaking into many pieces:

$$\Omega_{\text{patch}}(t) \quad \Longrightarrow \quad \bigcup_j A_j(t).$$

If the pieces collectively preserve positive stretching, then the route exits to

$$R_{\text{frag}}.$$

This is not immediate control. It means the aligned-patch estimate has failed into a later ordinary channel. The key diagnostic is an increasing effective positive component count:

$$N_{\text{eff}}^{+}(t) = \left(\sum_j (\pi_j^{+}(t))^2 \right)^{-1}.$$

If

$$N_{\text{eff}}^{+}(t)$$

becomes large while the total positive stretching remains significant, the patch is no longer a single coherent aligned support. It has become a fragmentation problem.

4.11 Scale-local exit route

A patch may become visible only after scale decomposition. This may happen if the support becomes thin, nested, filamentary, or broad-but-weak in a way that full-field diagnostics do not resolve.

Let

$$G_\ell$$

be a filter at scale

$$\ell \in \mathcal{L}.$$

If the positive stretching is organized only at a filtered scale, then the route exits to

$$R_{\text{scale}}.$$

This does not mean the contribution is controlled. It means that aligned-patch control has become scale-local control.

A theorem-level patch estimate should therefore include a scale-exit alternative:

$$R^+_{\text{patch}} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad R_{\text{scale}}.$$

4.12 Threshold or complement exit route

An aligned patch may cross a high-vorticity threshold or lie partly outside the selected high-vorticity region. Let

$$\Omega_\kappa(t) = \{x \in \Omega : |\omega(x,t)| > \kappa(t)\}.$$

If the stretching-active part of the patch lies in

$$\Omega \setminus \Omega_\kappa(t),$$

or flickers across the threshold, then the route may activate

$$R_{\text{low}}.$$

This prevents the aligned-patch channel from depending on an arbitrary cutoff. If the same positive stretching is missed by the high-vorticity mask, it must still appear in the complement channel.

4.13 Residual pathological exit route

If a patch-like route preserves positive stretching while avoiding aligned-patch control, transition-layer classification, fragmentation, scale-local visibility, and threshold/complement assignment, then it becomes residual:

$$R_{\text{path}}.$$

This is not a preferred outcome. It means the aligned-patch control attempt has exposed a residual pathological route. Paper 150I and later bridge work must then ask whether that route reduces to ordinary channels or becomes absorbable.

4.14 Conditional aligned-patch control principle

The aligned-patch control principle can now be stated.

Hypothesis 1 (Aligned-patch control alternative)**.** Let $R^+_{\mathrm{patch}}(I)$ be active on a smooth interval

$$I = [t_0, t_1].$$

Then at least one of the following alternatives holds:

1. **Pointwise absorbability:**

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t)$$

   on I;

2. **Integrated absorbability:**

$$\int_{t_0}^{t} R^+_{\mathrm{patch}}(s)\ \mathrm{d}s \le \delta_{\mathrm{patch}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\mathrm{patch}} \int_{t_0}^{t} E_\omega(s)\ \mathrm{d}s + C^0_{\mathrm{patch}}$$

   for every $t \in I$;

3. **Finite lifetime:** the patch persists for less than the characteristic stretching time needed to generate dangerous growth;

4. **Alignment loss:** the positive-stretching reservoir on the patch becomes lower-order;

5. **Transition-layer exit:**

$$R_{\mathrm{trans}}(I)$$

   activates;

6. **Fragmentation exit:**

$$R_{\mathrm{frag}}(I)$$

   activates;

7. **Scale-local exit:**

$$R_{\mathrm{scale}}(I)$$

   activates;

8. **Threshold/complement exit:**

$$R_{\mathrm{low}}(I)$$

   activates;

9. **Residual pathological exit:**

$$R_{\mathrm{path}}(I)$$

   activates.

This hypothesis is not yet a proof. It is the bridge target for the aligned-patch channel.

4.15 Margin condition for aligned-patch control

If the aligned-patch channel is controlled with coefficient

$$\delta_{\text{patch}},$$

then the final assembly must still satisfy

$$\theta + \delta_{\text{patch}} + \delta_{\text{trans}} + \delta_{\text{remaining}} < 1.$$

Thus aligned-patch control is useful only if

$$\delta_{\text{patch}}$$

does not consume the full dissipation reserve.

If the best available estimate gives

$$\theta + \delta_{\text{patch}} \geq 1,$$

then the patch is visible and estimated, but not controlled strongly enough for the Paper 150J closure.

A stronger target is to leave a positive reserve for downstream channels:

$$\delta_{\text{patch}} \leq (1 - \theta) - \delta_{\text{trans}} - \delta_{\text{reserve}},$$

where

$$\delta_{\text{reserve}} > 0$$

is reserved for fragmentation, scale-local transfer, complement behavior, and pathological concentration.

4.16 What this section proves and does not prove

This section formulates the aligned-patch control target. It does not prove that every aligned patch is absorbable. It does not prove that every coherent patch loses alignment, leaks, fragments, becomes scale-local, or enters the complement.

It states the required bridge alternative:

$$R^{+}_{\text{patch}} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad \text{finite lifetime} \quad \text{or} \quad \text{named exit route.}$$

This is the aligned-patch analogue of the visibility-to-control step needed after Paper 150L.

4.17 Summary

The aligned-patch channel is difficult because coherent strain alignment can preserve positive stretching while delaying directional-gradient cost. The control target is

$$R^{+}_{\text{patch}}(t) \leq \delta_{\text{patch}} D(t) + C_{\text{patch}} E_{\omega}(t),$$

or a subinterval-stable integrated analogue.

If absorbability fails, the patch must have finite lifetime, lose alignment, leak into a transition layer, fragment, become scale-local, enter the threshold/complement channel, or become residual pathology. The section also clarified that nonlocal strain may require multi-patch control and that the aligned-patch coefficient must leave reserve for downstream channels. The next section develops the geometry of the transition-layer channel.

5 Transition-Layer Geometry

The previous section formulated control targets for the aligned-patch channel. This section develops the geometry of the transition-layer channel

$$R_{\mathrm{trans}}.$$

The transition-layer channel is activated when dangerous positive stretching remains concentrated in a coherent core while a surrounding boundary or transition region mediates leakage, deformation, penetration, or loss of alignment.

This channel is distinct from the aligned-patch channel. An aligned patch is primarily a coherent support. A transition-layer structure has at least two regions: a stretching-active core and a surrounding layer where the patch interacts with the rest of the flow.

5.1 Core and transition region

Let

$$\Omega_{\mathrm{core}}(t) \subset \Omega$$

denote a stretching-active core. Let

$$\Omega_{\mathrm{trans}}(t) \subset \Omega$$

denote a surrounding transition region. The combined structure is

$$\Omega_{\mathrm{core}}(t) \cup \Omega_{\mathrm{trans}}(t).$$

The core carries positive stretching:

$$P^+_{\mathrm{core}}(t) = \int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V.$$

The transition region carries the interface between the core and the surrounding flow. It may contain magnitude gradients, directional gradients, leakage, shear, deformation, or partial loss of alignment.

The transition-layer channel becomes relevant when

$$P^+_{\mathrm{core}}(t)$$

remains significant while the boundary region controls how long the core can preserve stretching.

5.2 Transition layer as delayed depletion

A transition layer can delay depletion. A coherent core may continue to support positive stretching while the surrounding layer absorbs deformation or limits interaction with the outside flow. In that case, the core is not immediately controlled by directional-gradient cost, but the full structure is no longer a pure aligned patch.

The schematic geometry is:

$$\text{stretching-active core} \quad + \quad \text{transition layer.}$$

The transition layer may delay:

- leakage of vorticity magnitude out of the core;
- loss of vorticity-direction coherence;
- reduction of positive strain alignment;
- fragmentation of the core;
- interaction with surrounding strain fields.

Thus a transition layer is a protective mechanism, but it is also a possible cost-producing mechanism.

Remark 1 (Transition layers are not horizons). The word "protected" is used here in a strictly fluid-mechanical sense. A transition layer is not a horizon or a separate causal boundary. It is a region in which gradients, leakage, deformation, or alignment loss mediate the interaction between a stretching-active core and the surrounding flow. Its relevance is analytic: if the layer delays depletion, then it must either pay gradient or interface cost, leak, fragment, become scale-local, enter the complement, or become residual pathology. No gravitational or spacetime analogy is assumed.

5.3 Positive stretching in the core

The core is dangerous only if it carries significant positive stretching:

$$P^+_{\text{core}}(t) = \int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V.$$

A core with high vorticity but weak or negative strain alignment is not a transition-layer obstruction.

The relevant fraction of positive stretching carried by the core may be measured by

$$I^+_{\text{core}}(t) = \frac{\int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_{\Omega_{\text{core}}(t)\cup\Omega_{\text{trans}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V + \varepsilon}.$$

If

$$I^+_{\text{core}}(t) \approx 1,$$

then most of the positive stretching is still carried by the core. If this remains true while the transition layer prevents rapid depletion, then

$$R_{\text{trans}}$$

is active.

5.4 Transition-layer cost

The transition layer may pay cost through magnitude gradients, directional gradients, or interface deformation. A schematic transition-layer cost is

$$\mathcal{C}_{\mathrm{trans}}(t) = \nu \int_{\Omega_{\mathrm{trans}}(t)} |\nabla \omega|^2 \, \mathrm{d}V.$$

Using the gradient decomposition,

$$|\nabla \omega|^2 = |\nabla |\omega||^2 + |\omega|^2 |\nabla n|^2,$$

this becomes

$$\mathcal{C}_{\mathrm{trans}}(t) = \nu \int_{\Omega_{\mathrm{trans}}(t)} |\nabla |\omega||^2 \, \mathrm{d}V + \nu \int_{\Omega_{\mathrm{trans}}(t)} |\omega|^2 |\nabla n|^2 \, \mathrm{d}V.$$

The first term measures magnitude-gradient cost. The second term measures directional-gradient cost. A transition layer is favorable for control if it forces enough of this cost to absorb the core positive stretching.

A schematic target is:

$$P^+_{\mathrm{core}}(t) \le c_{\mathrm{trans}} \mathcal{C}_{\mathrm{trans}}(t) + C_{\mathrm{trans}} E_\omega(t).$$

If such an estimate holds with a small enough coefficient, the transition-layer channel becomes absorbable.

5.5 Leakage

A transition layer may also control the core through leakage. Leakage means that vorticity magnitude, directional coherence, or positive strain alignment escapes the core into the surrounding layer.

A schematic leakage diagnostic is

$$\mathcal{L}_{\mathrm{leak}}(t) = \frac{\int_{\Omega_{\mathrm{trans}}(t)} |\omega|^2 a_+(x,t) \, \mathrm{d}V}{\int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+(x,t) \, \mathrm{d}V + \varepsilon}.$$

Small

$$\mathcal{L}_{\mathrm{leak}}(t)$$

means the core remains protected. Large or increasing

$$\mathcal{L}_{\mathrm{leak}}(t)$$

means the core is losing isolation.

If leakage grows, then one of several outcomes may occur:

- the transition layer pays enough gradient cost for absorption;
- the core loses positive alignment;
- the structure fragments;

- the structure becomes scale-local;
- the route exits to residual pathology.

Thus leakage is both a sign of loss of protection and a possible route toward control.

5.6 Mask robustness of leakage diagnostics

The leakage diagnostic

$$\mathcal{L}_{\mathrm{leak}}(t) = \frac{\int_{\Omega_{\mathrm{trans}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V + \varepsilon}$$

depends on the choice of core and transition-layer masks. For sharp masks, a small shift of the boundary may change the measured leakage. This is a diagnostic risk, especially for diffuse or broad transition layers.

To reduce this dependence, one may use smooth weights

$$\chi_{\mathrm{core}}(x,t), \qquad \chi_{\mathrm{trans}}(x,t),$$

with

$$0 \le \chi_{\mathrm{core}}, \chi_{\mathrm{trans}} \le 1,$$

and

$$\chi_{\mathrm{core}} + \chi_{\mathrm{trans}} \le 1$$

on the relevant channel support, or a partition-of-unity version if all channel weights are included. A weighted leakage diagnostic is then

$$\mathcal{L}_{\mathrm{leak}\chi}(t) = \frac{\int_{\Omega} \chi_{\mathrm{trans}}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_{\Omega} \chi_{\mathrm{core}}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V + \varepsilon}.$$

A robust leakage claim should be stable under a declared class of smooth mask perturbations. For example, if

$$\chi_{\mathrm{core}}^{(\rho)}$$

denotes a family of smoothed core weights indexed by a smoothing or boundary-thickness parameter ρ, then leakage is robust only if the qualitative conclusion does not change under small admissible changes of ρ.

Thus leakage control should not depend on a jagged or arbitrary mask boundary. The analytic target is a leakage or transition-cost estimate that survives smooth core-layer perturbations. If the conclusion changes under small mask shifts, then the result is diagnostic rather than theorem-level control.

5.7 Core lifetime

A protected core is dangerous only if it lasts long enough to contribute to enstrophy growth. Let

$$I = [t_0, t_1]$$

be a smooth interval. The core lifetime may be measured by the set of times for which

$$P^+_{\mathrm{core}}(t)$$

remains significant and

$$I^+_{\mathrm{core}}(t)$$

remains large.

A schematic persistence condition is:

$$\left|\left\{t \in I : P^+_{\mathrm{core}}(t) \geq \eta(D(t) + E_\omega(t) + \varepsilon)\right\}\right| > 0.$$

A short-lived core may be harmless. A long-lived protected core is a transition-layer obstruction unless it pays enough cost, leaks, fragments, or loses alignment.

5.8 Protected core versus aligned patch

The aligned-patch and transition-layer channels overlap conceptually, but they represent different geometric states.

An aligned patch is primarily described by coherent support:

$$\Omega_{\mathrm{patch}}(t).$$

A transition-layer structure is described by a core and boundary region:

$$\Omega_{\mathrm{core}}(t) \cup \Omega_{\mathrm{trans}}(t).$$

A coherent aligned patch may become a transition-layer structure when a boundary becomes dynamically important. This transition can be written schematically as

$$R^+_{\mathrm{patch}} \quad \Longrightarrow \quad R_{\mathrm{trans}}.$$

The route has not yet been controlled. It has changed classification.

The key distinction is:

$$\text{aligned patch} = \text{coherent positive-stretching support},$$

while

$$\text{transition layer} = \text{protected core plus boundary/interfacial dynamics}.$$

5.9 Broad transition layers

A transition layer need not surround a compact hotspot. A broad coherent region may have diffuse boundaries or gradual transition zones. In that case, the transition layer may be thick, weak, or spatially extended.

This case is important for wide-area low-intensity danger. A broad region with weak positive alignment may be protected by gradual strain variation rather than a sharp boundary. If the broad support remains coherent, it may belong to

$$R^+_{\text{patch}}.$$

If a boundary or transition band controls leakage, alignment loss, or interaction with surrounding flow, it belongs to

$$R_{\text{trans}}.$$

If the broad support decomposes into many weak regions, it may exit to

$$R_{\text{frag}}$$

or

$$R_{\text{scale}}.$$

Thus transition-layer geometry includes both sharp and diffuse boundaries. For diffuse transition layers, hard masks should be treated as diagnostics, not as invariant objects. Smooth weights or coarse-grained transition bands are preferred when the boundary is not geometrically sharp.

5.10 Moving transition layers

A transition-layer structure may move through the domain. Fixed spatial masks may fail to detect its persistence. Therefore, transition layers should be allowed to be material, moving, weighted, or stopping-time localized.

A moving transition-layer family

$$\Omega_{\text{core}}(t) \cup \Omega_{\text{trans}}(t)$$

is still a transition-layer structure if:

- the core carries significant positive stretching;
- the surrounding layer mediates leakage, deformation, or alignment loss;
- the combined structure is trackable over time or over stopping-time intervals;
- the transition layer remains distinct from pure fragmentation or residual pathology.

If the moving structure cannot be tracked but still carries significant positive stretching, it may exit to

$$R_{\text{path}}.$$

5.11 Transition layer and nonlocal strain

The strain tensor

$$S_{ij}$$

is nonlocal. A protected core may be sustained by strain alignment produced by remote vorticity or by global incompressibility constraints. This means the transition layer may not be explained by local geometry alone.

Paper 150M treats this as part of the transition-layer control problem. If nonlocal strain preserves positive stretching inside a core while the surrounding layer delays leakage or alignment loss, then the contribution belongs to

$$R_{\mathrm{trans}}.$$

A theorem-level estimate must then explain how long such nonlocal support can persist, whether it pays gradient cost, whether it leaks, or whether it exits to another channel.

Nonlocal strain is therefore not a loophole. It is part of the analytic burden of controlling

$$R_{\mathrm{trans}}.$$

5.12 Definition of transition-layer structure

We now give the working definition.

Definition 2 (Transition-layer structure)**.** Let u be a smooth solution on

$$I = [t_0, t_1].$$

A family of sets

$$\Omega_{\mathrm{core}}(t) \subset \Omega, \qquad \Omega_{\mathrm{trans}}(t) \subset \Omega$$

defines a transition-layer structure on I if:

1. the core carries significant positive stretching:

$$\int_I \int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V\ \mathrm{d}t$$

   is non-negligible relative to the channel budget;

2. the transition region surrounds, borders, filters, or mediates the interaction between the core and the surrounding flow;

3. the transition region carries leakage, magnitude-gradient cost, directional-gradient cost, deformation, or alignment-loss diagnostics;

4. the combined core-layer structure is fixed, moving, material, weighted, or stopping-time trackable;

5. for diffuse or broad layers, the core-layer assignment is robust under admissible smooth mask perturbations, or else the claim is treated as diagnostic rather than theorem-level;

6. the contribution is not already assigned entirely to primary entry, pure aligned-patch support, fragmentation, scale-local transfer, low-vorticity complement stretching, or residual pathology.

This definition is intentionally flexible. It includes sharp boundary layers, diffuse transition bands, moving transition layers, and broad protected regions.

5.13 Transition-layer channel contribution

The transition-layer channel contribution is the part of the remainder assigned to protected core-layer structures:

$$R_{\text{trans}}(t).$$

In a partition-of-unity formulation, one may write schematically

$$R_{\text{trans}}(t) = \int_{\Omega} \chi_{\text{trans}}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V,$$

where

$$0 \le \chi_{\text{trans}} \le 1$$

is a measurable channel weight supported on transition-layer visibility.

The exact diagnostic for

$$\chi_{\text{trans}}$$

depends on how the core and layer are identified. Paper 150K supplies the accounting framework: if the transition-layer weight overlaps with other channel weights, the overlap must be charged explicitly.

5.14 Summary

A transition-layer structure consists of a stretching-active core and a surrounding or mediating layer. The core preserves positive stretching; the layer controls leakage, deformation, gradient cost, alignment loss, or interaction with the surrounding flow.

The transition-layer channel is difficult because the layer may protect the core before it pays enough cost. This protection is meant only in a fluid-mechanical sense: transition layers are interface and leakage structures, not horizons. The section also clarified that leakage diagnostics should be robust under smooth core-layer mask perturbations, especially for broad or diffuse transition layers. The next section formulates control targets for

$$R_{\text{trans}}.$$

6 Transition-Layer Control Targets

The previous section defined transition-layer geometry. This section formulates the control targets for the transition-layer channel

$$R_{\text{trans}}.$$

The transition-layer channel is difficult because a stretching-active core may remain coherent while a surrounding layer delays leakage, deformation, fragmentation, or loss of alignment. The layer may eventually produce cost, but the timing, robustness, and strength of that cost are the key analytic issues.

The goal of this section is not to prove unconditional transition-layer control. The goal is to state what must be shown for

$$R_{\text{trans}}$$

to become absorbable in the Paper 150J assembly, or else to identify the named exit routes when absorption fails.

6.1 The transition-layer obstruction

A transition-layer structure consists of a stretching-active core

$$\Omega_{\mathrm{core}}(t)$$

and a surrounding or mediating layer

$$\Omega_{\mathrm{trans}}(t).$$

The core contribution is

$$P^{+}_{\mathrm{core}}(t) = \int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_{+}(x,t)\ \mathrm{d}V.$$

The transition-layer channel is obstructive when this core contribution remains significant while the layer delays the normal routes to depletion.

The difficult regime is:

$$P^{+}_{\mathrm{core}}(t) \quad \text{large},$$

while the transition layer has not yet produced enough cost through

$$|\nabla|\omega||^2, \qquad |\omega|^2|\nabla n|^2,$$

or through leakage, deformation, fragmentation, scale-local transfer, or loss of positive alignment.

Thus R_{trans} represents protected positive stretching. It is visible, but not yet controlled.

6.2 Pointwise transition-layer target

The strongest useful target is a pointwise estimate:

$$R_{\mathrm{trans}}(t) \le \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_{\omega}(t).$$

Here

$$\delta_{\mathrm{trans}} \ge 0$$

is the dissipation fraction consumed by the transition-layer channel, and

$$C_{\mathrm{trans}} \ge 0$$

is a lower-order enstrophy coefficient.

This estimate is useful for closure only if the final Paper 150J margin remains positive:

$$\theta + \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}} + \delta_{\mathrm{remaining}} < 1.$$

Therefore, transition-layer control must be sharp. A bound with too large a dissipation coefficient identifies the channel but does not close the estimate.

6.3 Integrated transition-layer target

Transition layers may be moving, intermittent, or slowly leaking. A pointwise estimate may be too strong in such cases. The natural target is then a subinterval-stable integrated estimate:

$$\int_{t_0}^{t} R_{\mathrm{trans}}(s)\ \mathrm{d}s \leq \delta_{\mathrm{trans}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\mathrm{trans}} \int_{t_0}^{t} E_{\omega}(s)\ \mathrm{d}s + C^{0}_{\mathrm{trans}}$$

for every

$$t \in I.$$

The phrase "for every $t \in I$" is essential. A full-interval estimate alone may hide a transient spike in the protected core. Paper 150K showed that integrated estimates support closure only when they are stable on all subintervals, or when a stopping-time partial-sum structure controls all intermediate times.

6.4 Transition-layer cost route

The most direct control route is transition-layer cost. The layer may absorb the core contribution through magnitude-gradient and directional-gradient dissipation:

$$\mathcal{C}_{\mathrm{trans}}(t) = \nu \int_{\Omega_{\mathrm{trans}}(t)} |\nabla \omega|^2\ \mathrm{d}V.$$

Using

$$|\nabla \omega|^2 = |\nabla |\omega||^2 + |\omega|^2 |\nabla n|^2,$$

we have

$$\mathcal{C}_{\mathrm{trans}}(t) = \nu \int_{\Omega_{\mathrm{trans}}(t)} |\nabla |\omega||^2\ \mathrm{d}V + \nu \int_{\Omega_{\mathrm{trans}}(t)} |\omega|^2 |\nabla n|^2\ \mathrm{d}V.$$

A schematic transition-cost estimate is

$$P^{+}_{\mathrm{core}}(t) \leq c_{\mathrm{trans}} \mathcal{C}_{\mathrm{trans}}(t) + C_{\mathrm{trans}} E_{\omega}(t).$$

If this estimate holds with a small enough effective coefficient, then the protected core becomes absorbable through the cost paid by its transition layer.

The difficult case is a core that remains protected while the transition-layer cost stays too small.

6.5 Leakage route

A transition layer may control the core by leakage. Leakage means that the core loses isolation: vorticity magnitude, directional coherence, or positive strain alignment begins to move into the surrounding layer.

Recall the leakage diagnostic

$$\mathcal{L}_{\mathrm{leak}}(t) = \frac{\int_{\Omega_{\mathrm{trans}}(t)} |\omega|^2 a_{+}(x,t)\ \mathrm{d}V}{\int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_{+}(x,t)\ \mathrm{d}V + \varepsilon}.$$

If

$$\mathcal{L}_{\text{leak}}(t)$$

increases, then the protected core is no longer isolated. This may lead to several outcomes:

- the transition layer pays enough gradient cost for absorption;
- positive stretching spreads and becomes lower-order;
- the structure fragments into R_{frag};
- the structure becomes scale-local and exits to R_{scale};
- the route becomes residual and exits to R_{path}.

A leakage-based control route would show that a core cannot preserve significant positive stretching for long unless either leakage remains small at real cost or leakage grows and pushes the structure into a controlled or named exit channel.

6.6 Mask-robust leakage route

Leakage control should not depend on an arbitrary choice of core and transition-layer masks. For diffuse or broad transition layers, a small shift of the core boundary may change the value of

$$\mathcal{L}_{\text{leak}}(t).$$

A theorem-level leakage route should therefore be robust under a declared class of smooth mask perturbations.

Let

$$\chi^{(\rho)}_{\text{core}}(x,t), \qquad \chi^{(\rho)}_{\text{trans}}(x,t)$$

be a family of smooth core and transition weights indexed by a boundary-thickness or smoothing parameter

$$\rho.$$

A weighted leakage diagnostic is

$$\mathcal{L}_{\text{leak}}{}^{(\rho)}_{\chi}(t) = \frac{\int_\Omega \chi^{(\rho)}_{\text{trans}}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_\Omega \chi^{(\rho)}_{\text{core}}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V + \varepsilon}.$$

A leakage-based estimate is robust only if its qualitative conclusion is stable for admissible variations of

$$\rho.$$

If small smooth changes of the mask reverse the conclusion, then the leakage measurement is diagnostic rather than theorem-level control.

Thus the transition-layer control target is not merely to prove leakage for one chosen mask. It is to prove leakage, transition-layer cost, or exit-channel activation in a way that survives smooth core-layer perturbations.

6.7 Core-lifetime route

A protected core is dangerous only if it persists long enough. A short-lived protected core may produce little integrated positive stretching even if its instantaneous stretching is large.

A natural comparison time is the local stretching time

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty(\Omega_{\text{core}}(t)\cup\Omega_{\text{trans}}(t))} + \varepsilon},$$

or the local positive-alignment time

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty(\Omega_{\text{core}}(t)\cup\Omega_{\text{trans}}(t))} + \varepsilon}.$$

If the protected core survives for many such times while keeping

$$P^+_{\text{core}}(t)$$

large, then it becomes a serious transition-layer obstruction.

A core-lifetime control target is to show that the time set on which

$$P^+_{\text{core}}(t)$$

is significant and

$$I^+_{\text{core}}(t) \approx 1$$

has controlled measure, unless the transition layer pays cost or an exit channel activates.

A schematic lifetime estimate is

$$\int_I P^+_{\text{core}}(t)\ \mathrm{d}t \le c_{\text{life}} \int_I \mathcal{C}_{\text{trans}}(t)\ \mathrm{d}t + C_{\text{life}} \int_I E_\omega(t)\ \mathrm{d}t + C^0_{\text{life}}.$$

Such an estimate would say that a long-lived protected core must be paid for by transition-layer cost or lower-order enstrophy.

6.8 Alignment-loss route

A transition-layer structure may stop being dangerous if the core loses positive strain alignment. The relevant quantity is

$$a_+(x,t) = \max\{n_i S_{ij} n_j, 0\}.$$

If the core remains high-vorticity but

$$a_+(x,t)$$

drops, then the positive stretching reservoir decreases.

A schematic core-alignment fraction is

$$\Pi^+_{\text{core}}(t) = \frac{\int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_{\Omega_{\text{core}}(t)} |\omega|^2 |a(x,t)|\ \mathrm{d}V + \varepsilon}.$$

If

$$\Pi^+_{\mathrm{core}}(t)$$

falls below a declared threshold, then the core is no longer dominantly stretching-active. In that case, the transition-layer contribution may become lower-order or exit to another channel.

This route reduces the positive-stretching reservoir rather than absorbing it directly through dissipation.

6.9 Fragmentation exit route

A transition-layer structure may fail by breaking into multiple stretching-active pieces:

$$\Omega_{\mathrm{core}}(t) \cup \Omega_{\mathrm{trans}}(t) \quad \Longrightarrow \quad \bigcup_j A_j(t).$$

If those pieces collectively carry positive stretching, then the route exits to

$$R_{\mathrm{frag}}.$$

The fragmentation diagnostic may again use an effective positive component count:

$$N^+_{\mathrm{eff}}(t) = \left(\sum_j (\pi^+_j(t))^2 \right)^{-1}.$$

If

$$N^+_{\mathrm{eff}}(t)$$

increases while the total positive stretching remains significant, then the protected core-layer picture has broken down into a fragmentation problem.

This is not closure by itself. It is a named exit route for later ordinary-channel control.

6.10 Scale-local exit route

A transition layer may become thin, diffuse, nested, or visible only at a particular scale. In that case, the structure may exit to

$$R_{\mathrm{scale}}.$$

Let

$$G_\ell$$

be a filter at scale

$$\ell \in \mathcal{L}.$$

If the core-layer geometry is detected only after filtering, then the correct classification may be scale-local rather than transition-layer.

A theorem-level transition-layer estimate should therefore include a scale-local exit:

$$R_{\mathrm{trans}} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad R_{\mathrm{scale}}.$$

6.11 Threshold or complement exit route

A transition-layer structure may cross a high-vorticity threshold or occupy the complement of the selected high-vorticity region. Let

$$\Omega_\kappa(t) = \{x \in \Omega : |\omega(x,t)| > \kappa(t)\}.$$

If the stretching-active core or transition layer lies partly outside

$$\Omega_\kappa(t),$$

then the contribution may activate

$$R_{\text{low}}.$$

If the structure flickers across the threshold, the relevant question is whether the cumulative positive stretching remains significant. If it does, the route must be assigned to

$$R_{\text{low}}, \quad R_{\text{scale}}, \quad \text{or} \quad R_{\text{path}},$$

depending on whether complement, scale, or residual diagnostics capture it.

Thus threshold motion is not a loophole. It is a channel transition.

6.12 Residual pathological exit route

If a transition-layer route preserves positive stretching while avoiding transition-layer cost, leakage, alignment loss, fragmentation, scale-local classification, and threshold/complement classification, then it becomes residual:

$$R_{\text{path}}.$$

This is the failure mode in which a protected core-layer structure remains dangerous but cannot be controlled or reclassified by ordinary means. Paper 150I and later bridge work must then ask whether the residual route reduces to ordinary channels or becomes absorbable.

6.13 Conditional transition-layer control principle

The transition-layer control principle can now be stated.

Hypothesis 2 (Transition-layer control alternative)**.** Let $R_{\text{trans}}(I)$ be active on a smooth interval

$$I = [t_0, t_1].$$

Then at least one of the following alternatives holds:

1. **Pointwise absorbability:**

   $$R_{\text{trans}}(t) \le \delta_{\text{trans}} D(t) + C_{\text{trans}} E_\omega(t)$$

   on I;

2. **Integrated absorbability:**

$$\int_{t_0}^{t} R_{\text{trans}}(s)\ \mathrm{d}s \le \delta_{\text{trans}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\text{trans}} \int_{t_0}^{t} E_{\omega}(s)\ \mathrm{d}s + C^0_{\text{trans}}$$

   for every $t \in I$;

3. **Transition-layer cost:** the layer pays enough magnitude-gradient, directional-gradient, or interface cost to absorb the core contribution;

4. **Mask-robust leakage:** the core loses isolation under a smooth, robust core-layer diagnostic and the route becomes lower-order or exits to another channel;

5. **Finite lifetime:** the core persists for less than the characteristic stretching time needed to generate dangerous growth;

6. **Alignment loss:** the core no longer carries dominantly positive stretching;

7. **Fragmentation exit:**

$$R_{\text{frag}}(I)$$

   activates;

8. **Scale-local exit:**

$$R_{\text{scale}}(I)$$

   activates;

9. **Threshold/complement exit:**

$$R_{\text{low}}(I)$$

   activates;

10. **Residual pathological exit:**

$$R_{\text{path}}(I)$$

   activates.

This hypothesis is not yet a proof. It is the bridge target for the transition-layer channel.

6.14 Margin condition for transition-layer control

If the transition-layer channel is controlled with coefficient

$$\delta_{\text{trans}},$$

then the final assembly must satisfy

$$\theta + \delta_{\text{patch}} + \delta_{\text{trans}} + \delta_{\text{remaining}} < 1.$$

Thus transition-layer control is useful only if

$$\delta_{\text{trans}}$$

does not consume the remaining dissipation reserve.

If the best available estimate gives

$$\theta + \delta_{\text{patch}} + \delta_{\text{trans}} \geq 1,$$

then the transition-layer contribution is visible and estimated, but the estimate is too costly to close the Paper 150J margin.

A stronger target is to preserve a positive reserve for downstream channels:

$$\delta_{\text{trans}} \leq (1 - \theta) - \delta_{\text{patch}} - \delta_{\text{reserve}},$$

where

$$\delta_{\text{reserve}} > 0$$

is reserved for fragmentation, scale-local transfer, complement behavior, and pathological concentration.

6.15 What this section proves and does not prove

This section formulates the transition-layer control target. It does not prove that every transition-layer structure is absorbable. It does not prove that every protected core leaks, fragments, loses alignment, becomes scale-local, or enters the complement.

It states the required bridge alternative:

$$R_{\text{trans}} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad \text{finite lifetime} \quad \text{or} \quad \text{named exit route.}$$

This is the transition-layer analogue of the visibility-to-control step needed after Paper 150L.

6.16 Summary

The transition-layer channel is difficult because a protected core may preserve positive stretching while the layer delays depletion. The control target is

$$R_{\text{trans}}(t) \leq \delta_{\text{trans}} D(t) + C_{\text{trans}} E_{\omega}(t),$$

or a subinterval-stable integrated analogue.

If absorbability fails, the structure must pay transition-layer cost, leak in a mask-robust way, have finite lifetime, lose alignment, fragment, become scale-local, enter the threshold/complement channel, or become residual pathology. The next section studies patch-to-transition failure modes and channel transitions.

7 Patch-to-Transition Failure Modes

The previous sections formulated the aligned-patch and transition-layer control targets. This section studies how those targets can fail and how failure should be classified. The purpose is to keep the coherent-channel program from treating every difficult case as the same obstruction.

A coherent aligned patch may be controlled directly, or it may fail by leaking into a transition layer. A transition layer may then be controlled directly, or it may fail by fragmenting, becoming scale-local, entering a threshold/complement channel, or becoming residual pathology. These are not vague failures. They are named channel transitions.

7.1 Why patch failure needs classification

The aligned-patch channel is active when coherent support carries significant positive stretching:

$$R^+_{\mathrm{patch}}(t) \sim \int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+(x,t)\, \mathrm{d}V.$$

The desired estimate is

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t),$$

or its subinterval-stable integrated analogue.

If this estimate fails, the failure should not remain undefined. A persistent coherent patch must do one of the following:

- remain coherent and aligned with too little cost;
- persist across many stretching times;
- leak into a transition layer;
- lose positive alignment;
- fragment;
- become scale-local;
- cross into the threshold/complement channel;
- become residual pathology.

The first two cases are genuine aligned-patch obstructions. The other cases are channel transitions.

7.2 Coherent transport corridors

A coherent aligned patch or transition-layer structure may also be interpreted as a local coherent transport corridor in the classical fluid-mechanical sense. This means a connected or semi-connected region in which vorticity direction, strain alignment, and positive stretching remain organized over time.

This terminology does not introduce a new force or a non-Navier–Stokes mechanism. It only describes the geometry of transport inside the existing velocity and vorticity fields. A corridor is dangerous if it preserves positive stretching while delaying directional-gradient cost, leakage, fragmentation, or alignment loss.

In the channel language,

$$\text{coherent transport corridor} \quad \Longrightarrow \quad R^+_{\mathrm{patch}} \quad \text{or} \quad R_{\mathrm{trans}},$$

depending on whether the structure is better described as coherent support or as a protected core with a mediating transition layer.

7.3 Pure aligned-patch obstruction

A pure aligned-patch obstruction occurs when the patch preserves positive stretching while avoiding the known control routes. In schematic form,

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+ \, \mathrm{d}V$$

remains significant, while

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2 \, \mathrm{d}V$$

remains too small to absorb it, boundary cost remains too small, and the patch does not leak, fragment, become scale-local, or enter the complement.

This is the strongest aligned-patch failure. It says that coherent strain alignment can preserve stretching without paying enough visible cost. If such a route exists, then R^+_{patch} remains an independent obstruction.

A theorem-level control result must rule out this case or isolate it as a falsifier.

7.4 Persistence across stretching times

A pure aligned patch is especially dangerous if it persists across many local stretching times. Let

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty(\Omega_{\text{patch}}(t))} + \varepsilon}$$

or

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty(\Omega_{\text{patch}}(t))} + \varepsilon}$$

be characteristic stretching times on the patch. If the patch lifetime

$$\tau_{\text{patch}}$$

satisfies

$$\tau_{\text{patch}} \gg \tau_S$$

while positive stretching remains significant and directional-gradient cost remains small, then the patch has enough time to contribute materially to enstrophy growth.

This is a stronger version of the pure aligned-patch obstruction. It says not only that the patch is coherent, but that it remains coherent long enough to matter dynamically. A theorem-level control result must either bound this lifetime, force a cost, or classify the route as a downstream channel.

7.5 Leakage into transition layer

The most natural failure of pure patch coherence is leakage into a transition layer:

$$R^+_{\text{patch}} \quad \Longrightarrow \quad R_{\text{trans}}.$$

This occurs when the patch develops a core-layer structure:

$$\Omega_{\mathrm{core}}(t) \cup \Omega_{\mathrm{trans}}(t),$$

where the core still carries positive stretching but the surrounding layer begins to mediate leakage, deformation, magnitude-gradient cost, directional-gradient cost, or alignment loss.

A leakage diagnostic is

$$\mathcal{L}_{\mathrm{leak}}(t) = \frac{\int_{\Omega_{\mathrm{trans}}(t)} |\omega|^2 a_+ \, \mathrm{d}V}{\int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+ \, \mathrm{d}V + \varepsilon}.$$

If

$$\mathcal{L}_{\mathrm{leak}}(t)$$

increases while the core remains stretching-active, then the patch has not disappeared. It has changed classification.

This transition is useful because it narrows the later analytic burden. Instead of proving direct aligned-patch absorbability, one must prove transition-layer control.

7.6 Mask-sensitive leakage failure

For diffuse boundaries, leakage may depend on the choice of core and transition-layer masks. A mask-sensitive failure occurs if a patch appears to leak under one choice of

$$\chi_{\mathrm{core}}, \qquad \chi_{\mathrm{trans}},$$

but not under a nearby smooth perturbation of those weights.

This is not a theorem-level leakage result. It is a diagnostic ambiguity. A robust patch-to-transition classification should survive smooth changes in the boundary or weighting scheme, or else the route should be treated as unresolved and possibly scale-local or residual.

Thus a transition from

$$R^+_{\mathrm{patch}} \quad \text{to} \quad R_{\mathrm{trans}}$$

should be based on stable leakage, stable transition-layer cost, or stable core-layer geometry, not on a jagged or arbitrary mask boundary.

7.7 Patch alignment loss

A patch may fail to remain dangerous because it loses positive strain alignment. The aligned fraction

$$\Pi^+_{\mathrm{patch}}(t) = \frac{\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+ \, \mathrm{d}V}{\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 |a| \, \mathrm{d}V + \varepsilon}$$

may decrease. If

$$\Pi^+_{\mathrm{patch}}(t)$$

falls below a declared threshold, then the patch no longer carries dominantly positive stretching.

This is a favorable failure. It reduces the positive-stretching reservoir directly. The patch may remain high-vorticity, but it is no longer a strong aligned-patch obstruction if it is no longer positively aligned with strain.

7.8 Patch fragmentation

A coherent patch may break into many stretching-active pieces:

$$\Omega_{\text{patch}}(t) \quad \Longrightarrow \quad \bigcup_j A_j(t).$$

If the pieces collectively carry significant positive stretching, the route exits to

$$R_{\text{frag}}.$$

The effective positive component count

$$N_{\text{eff}}^{+}(t) = \left(\sum_j (\pi_j^{+}(t))^2 \right)^{-1}$$

is a useful diagnostic. If

$$N_{\text{eff}}^{+}(t)$$

increases while total positive stretching remains significant, then the original patch has become a fragmentation problem.

Fragmentation may eventually help control because many components can create interface cost, reduce coherent strain alignment, or distribute stretching in a way that becomes absorbable. However, Paper 150M does not prove that. It only classifies the exit route.

7.9 Patch scale-localization

A patch may become thin, filamentary, nested, broad-but-weak, or otherwise visible only after filtering. Then it exits to the scale-local channel:

$$R_{\text{scale}}.$$

Let

$$G_\ell$$

be a filter at scale

$$\ell \in \mathcal{L}.$$

If the positive stretching is organized only at some filtered scale, then the patch is no longer a pure aligned-patch object. It is scale-local.

This case is important because a patch may appear coherent at one scale and fragmented or diffuse at another. A theorem-level estimate must therefore state the scale family being used and account for any unresolved scale-local route.

7.10 Patch threshold or complement exit

A patch may cross a high-vorticity threshold or lie partly outside the selected high-vorticity set:

$$\Omega_\kappa(t) = \{x \in \Omega : |\omega(x,t)| > \kappa(t)\}.$$

If the stretching-active part of the patch lies in

$$\Omega \setminus \Omega_\kappa(t),$$

then the route exits to

$$R_{\text{low}}.$$

Threshold flicker is a related case. If the patch repeatedly moves in and out of

$$\Omega_\kappa(t)$$

while its cumulative positive stretching remains significant, then the route should be assigned to

$$R_{\text{low}}, \quad R_{\text{scale}}, \quad \text{or} \quad R_{\text{path}},$$

depending on whether complement, scale, or residual diagnostics capture it.

Thus threshold motion is not a way to hide from the taxonomy. It is a named exit route.

7.11 Transition-layer failure modes

Once a patch exits to

$$R_{\text{trans}},$$

the transition-layer channel has its own failure modes. The desired estimate is

$$R_{\text{trans}}(t) \le \delta_{\text{trans}} D(t) + C_{\text{trans}} E_\omega(t),$$

or a subinterval-stable integrated analogue.

If this estimate fails, then the transition-layer structure must do one of the following:

- preserve a protected core with too little transition-layer cost;
- leak enough to become lower-order or move to another channel;
- lose core alignment;
- fragment;
- become scale-local;
- enter the threshold/complement channel;
- become residual pathology.

The first case is the genuine transition-layer obstruction. The others are exits.

7.12 Protected-core obstruction

A protected-core obstruction occurs when

$$P_{\text{core}}^{+}(t) = \int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+ \, \mathrm{d}V$$

remains significant, while the transition-layer cost

$$\mathcal{C}_{\text{trans}}(t) = \nu \int_{\Omega_{\text{trans}}(t)} |\nabla\omega|^2 \, \mathrm{d}V$$

is too small to absorb it, leakage remains low, alignment remains positive, and the structure does not fragment, become scale-local, enter the complement, or become residual pathology.

This is the strongest transition-layer failure. It says that a protected core can preserve stretching without paying enough transition-layer cost. If such a route exists, then R_{trans} remains an independent obstruction.

7.13 Nonlocal feedback failure

A coherent patch or protected core may be sustained by nonlocal strain feedback from another region. For example, two separated patches

$$\Omega_{\text{patch}}^{(1)}(t), \qquad \Omega_{\text{patch}}^{(2)}(t)$$

may contribute to the strain alignment experienced by each other.

A nonlocal feedback failure occurs if separated structures preserve positive stretching for one another while each local support pays too little local directional-gradient, boundary, or transition-layer cost. In such a case, neither support alone carries the full cost of the alignment it receives.

This does not invalidate the channel taxonomy. It identifies a missing estimate. The correct control object may be a coupled support:

$$\Omega_{\text{patch}}^{\text{coup}}(t) = \bigcup_m \Omega_{\text{patch}}^{(m)}(t),$$

which must either pay joint cost, fragment, become scale-local, enter the complement, or become residual pathology.

7.14 Transition-layer fragmentation

A protected core-layer structure may break into many pieces:

$$\Omega_{\text{core}}(t) \cup \Omega_{\text{trans}}(t) \quad \Longrightarrow \quad \bigcup_j A_j(t).$$

If the pieces collectively carry significant positive stretching, the route exits to

$$R_{\text{frag}}.$$

This is no longer a transition-layer-only problem. It becomes a fragmentation-control problem for later papers.

7.15 Transition-layer scale-localization

A transition layer may become thin, diffuse, or visible only through filtering. In that case, it exits to

$$R_{\text{scale}}.$$

A scale-local transition layer may still be dangerous, but it is no longer controlled solely by the transition-layer framework.

This exit is especially important for diffuse or broad transition bands. A layer may not have a sharp boundary, but it may become visible at a filtered scale.

7.16 Transition-layer threshold or complement exit

A protected core or transition layer may cross the chosen high-vorticity threshold. If a significant part of the positive stretching lies outside

$$\Omega_\kappa(t),$$

then the route exits to

$$R_{\text{low}}.$$

If the transition layer flickers across the threshold, the cumulative positive stretching must still be accounted for. The route must activate complement, scale-local, or residual pathological visibility.

7.17 Residual pathological failure

If a patch or transition-layer structure preserves significant positive stretching while avoiding all control routes and all ordinary exits, then it becomes residual:

$$R_{\text{path}}.$$

This case is not thrown away. It becomes a named obstruction:

$$\text{residual coherent-channel pathology.}$$

Paper 150I and later bridge work must then ask whether the route reduces to ordinary channels or becomes absorbable.

7.18 Channel transition map

The coherent-channel transition map is:

$$\begin{aligned}
R^+_{\text{patch}} \quad &\Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad R_{\text{trans}}, R_{\text{frag}}, R_{\text{scale}}, R_{\text{low}}, R_{\text{path}}, \\
R_{\text{trans}} \quad &\Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad R_{\text{frag}}, R_{\text{scale}}, R_{\text{low}}, R_{\text{path}}.
\end{aligned}$$

This map is the main output of the present section. It states that coherent-channel failure is not vague. It must become cost, transition, fragmentation, scale-localization, threshold/complement behavior, or residual pathology.

7.19 No double-counting during transitions

When a route transitions from one channel to another, the contribution should not be counted twice. For example, if a patch becomes a transition-layer structure, the contribution should move from

$$R^+_{\mathrm{patch}}$$

to

$$R_{\mathrm{trans}}$$

under a measurable partition or stopping-time reassignment.

Paper 150K supplies the accounting rule. If the channel weights form a partition of unity, the contribution is counted once. If the channel diagnostics overlap, the overlap must be charged explicitly. Thus a channel transition is a classification change, not a license to double-count stretching.

7.20 Budget dominance during transitions

Even when channel transitions are classified correctly, they may still exhaust the margin. A patch that exits to a transition layer may be controlled locally, but if the combined coefficient

$$\delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

is too large, then little or no budget remains for

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

Thus channel transitions must be tracked not only for classification, but also for coefficient cost. A successful patch-to-transition program must preserve a positive reserve for downstream channels.

7.21 Summary

Aligned-patch and transition-layer control can fail in named ways. A pure aligned-patch obstruction preserves positive stretching with too little directional or boundary cost. A persistent patch obstruction survives across many stretching times. A protected-core obstruction preserves stretching with too little transition-layer cost or leakage. Nonlocal feedback can couple separated coherent structures and may require joint multi-patch estimates.

Other failures exit to fragmentation, scale-local transfer, threshold/complement support, or residual pathology. Channel transitions must also avoid double-counting and must preserve enough dissipation budget for downstream channels.

The next section states the conditional ordinary-channel control theorem for

$$R^+_{\mathrm{patch}} \quad \text{and} \quad R_{\mathrm{trans}}.$$

8 Conditional Ordinary-Channel Control Theorem

The previous section described how aligned-patch and transition-layer control can fail into named channel transitions. This section assembles those alternatives into a conditional ordinary-channel control theorem for the two coherent channels studied in Paper 150M:

$$R^{+}_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

The theorem is conditional. It does not prove unconditional Navier–Stokes regularity. It states that if aligned-patch and transition-layer routes either become absorbable, have finite lifetime, or exit to named channels, then they do not remain independent coherent-channel obstructions to the Paper 150J assembly.

8.1 The coherent-channel target

Define the coherent-channel contribution

$$R_{\mathrm{coh}}(t) = R^{+}_{\mathrm{patch}}(t) + R_{\mathrm{trans}}(t).$$

The desired pointwise estimate is

$$R_{\mathrm{coh}}(t) \le \delta_{\mathrm{coh}} D(t) + C_{\mathrm{coh}} E_{\omega}(t),$$

where

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

and

$$C_{\mathrm{coh}} = C_{\mathrm{patch}} + C_{\mathrm{trans}}.$$

The corresponding integrated estimate is

$$\int_{t_0}^{t} R_{\mathrm{coh}}(s)\ \mathrm{d}s \le \delta_{\mathrm{coh}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\mathrm{coh}} \int_{t_0}^{t} E_{\omega}(s)\ \mathrm{d}s + C^{0}_{\mathrm{coh}}$$

for every

$$t \in I.$$

This subinterval-stability requirement is essential. It is the condition that prevents an integrated estimate from hiding a transient enstrophy spike.

8.2 Coherent-channel control alternatives

The coherent-channel control program rests on two broad alternatives.

First, the coherent channels may be directly absorbable:

$$R^{+}_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_{\omega}(t),$$

and

$$R_{\mathrm{trans}}(t) \le \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_{\omega}(t),$$

or corresponding subinterval-stable integrated estimates.

Second, if direct absorbability fails, the coherent route must either have finite lifetime or exit to a named channel:

$$R_{\rm frag}, \quad R_{\rm scale}, \quad R_{\rm low}, \quad R_{\rm path},$$

or, in the case of a pure aligned patch, first to

$$R_{\rm trans}.$$

Thus the coherent-channel target is not merely:

$$R^+_{\rm patch} + R_{\rm trans} \quad \text{is small.}$$

It is the weaker but more realistic statement:

$$R^+_{\rm patch} + R_{\rm trans} \quad \text{is absorbable, short-lived relative to stretching time, or exits to named channels.}$$

8.3 Hypotheses

The theorem uses the following hypotheses.

Hypothesis 3 (Aligned-patch alternative)**.** If

$$R^+_{\rm patch}(I)$$

is active on a smooth interval

$$I = [t_0, t_1],$$

then one of the alternatives in Hypothesis 1 holds: pointwise absorbability, integrated absorbability, finite lifetime, alignment loss, transition-layer exit, fragmentation exit, scale-local exit, threshold/complement exit, or residual pathological exit.

Hypothesis 4 (Transition-layer alternative)**.** If

$$R_{\rm trans}(I)$$

is active on a smooth interval

$$I = [t_0, t_1],$$

then one of the alternatives in Hypothesis 2 holds: pointwise absorbability, integrated absorbability, transition-layer cost, mask-robust leakage, finite lifetime, alignment loss, fragmentation exit, scale-local exit, threshold/complement exit, or residual pathological exit.

Hypothesis 5 (No double-counting under channel transitions)**.** If a coherent route exits from one channel to another, the corresponding positive-stretching contribution is reassigned through a measurable partition, bounded-overlap weight, stopping-time decomposition, or equivalent accounting device. The contribution is not counted simultaneously as both the original coherent channel and the exit channel, except through an explicitly charged overlap budget.

Hypothesis 6 (Multi-patch nonlocal strain assignment)**.** If coherent strain alignment is sustained by separated supports, then the contribution is assigned either to a coupled coherent support,

$$\Omega_{\mathrm{patch}}^{\mathrm{coup}}(t) = \bigcup_{m=1}^{M} \Omega_{\mathrm{patch}}^{(m)}(t),$$

or to a named downstream channel. The nonlocal contribution is not allowed to remain outside the channel accounting.

Hypothesis 7 (Coherent-channel margin compatibility)**.** The absorbable coherent-channel coefficients satisfy

$$\theta + \delta_{\mathrm{coh}} + \delta_{\mathrm{remaining}} < 1,$$

where

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

and

$$\delta_{\mathrm{remaining}} = \delta_{\mathrm{frag}} + \delta_{\mathrm{scale}} + \delta_{\mathrm{low}} + \delta_{\mathrm{path}}.$$

Moreover, the coherent-channel coefficient leaves a positive reserve for downstream channels:

$$\delta_{\mathrm{coh}} \leq (1-\theta) - \delta_{\mathrm{reserve}}, \qquad \delta_{\mathrm{reserve}} > 0,$$

where the reserve is intended for fragmentation, scale-local transfer, complement behavior, and pathological concentration.

The final hypothesis is the margin condition needed by the Paper 150J assembly. Without it, the channels may be visible and estimated but still too expensive to close the enstrophy inequality.

8.4 Conditional coherent-channel control theorem

Theorem 1 (Conditional coherent-channel control theorem)**.** *Let u be a smooth solution of the three-dimensional incompressible Navier–Stokes equations on*

$$I = [t_0, t_1].$$

Assume the aligned-patch alternative, the transition-layer alternative, no double-counting under channel transitions, multi-patch nonlocal strain assignment, and coherent-channel margin compatibility.

Then the coherent channels

$$R_{\mathrm{patch}}^{+} \qquad \textit{and} \qquad R_{\mathrm{trans}}$$

do not remain independent obstructions to the Paper 150J assembly. More precisely, each active coherent-channel contribution is either:

1. *absorbed by dissipation and lower-order enstrophy with coefficient included in*

$$\delta_{\mathrm{coh}},$$

2. shown to have finite lifetime relative to the characteristic stretching time, so that its integrated positive-stretching contribution is lower-order or assigned to an integrated estimate, or

3. reassigned to a named exit channel:

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad \textit{or} \quad R_{\mathrm{path}},$$

with no double-counting of the original coherent contribution.

Proof. Assume first that

$$R^+_{\mathrm{patch}}(I)$$

is active. By the aligned-patch alternative, one of the listed outcomes occurs.

If pointwise absorbability holds, then

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t).$$

If integrated absorbability holds, then

$$\int_{t_0}^t R^+_{\mathrm{patch}}(s)\ \mathrm{d}s \le \delta_{\mathrm{patch}} \int_{t_0}^t D(s)\ \mathrm{d}s + C_{\mathrm{patch}} \int_{t_0}^t E_\omega(s)\ \mathrm{d}s + C^0_{\mathrm{patch}}$$

for every $t \in I$. In either case, the patch contribution is absorbable in the sense required by the Paper 150J assembly, subject to the stated margin condition.

If finite lifetime holds, then the patch does not persist long enough relative to the characteristic stretching time to remain an independent coherent-channel obstruction. Its contribution is either lower-order or included in the appropriate integrated estimate.

If absorbability fails but alignment loss occurs, then the positive-stretching reservoir assigned to the patch becomes lower-order and no longer remains an independent aligned-patch obstruction.

If the route exits to

$$R_{\mathrm{trans}}, \quad R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad \text{or} \quad R_{\mathrm{path}},$$

then the no-double-counting hypothesis assigns the contribution to the corresponding exit channel. It is no longer counted independently as R^+_{patch}, except through any explicitly charged overlap budget.

Now assume

$$R_{\mathrm{trans}}(I)$$

is active. By the transition-layer alternative, one of the listed outcomes occurs.

If pointwise absorbability holds, then

$$R_{\mathrm{trans}}(t) \le \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_\omega(t).$$

If integrated absorbability holds, then

$$\int_{t_0}^t R_{\mathrm{trans}}(s)\ \mathrm{d}s \le \delta_{\mathrm{trans}} \int_{t_0}^t D(s)\ \mathrm{d}s + C_{\mathrm{trans}} \int_{t_0}^t E_\omega(s)\ \mathrm{d}s + C^0_{\mathrm{trans}}$$

for every $t \in I$. Thus the transition-layer contribution is absorbable in the required pointwise or integrated sense.

If transition-layer cost, mask-robust leakage, finite lifetime, or alignment loss reduces the positive-stretching reservoir to an absorbable or lower-order term, then R_{trans} no longer remains an independent obstruction.

If the route exits to

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad \text{or} \quad R_{\mathrm{path}},$$

then the no-double-counting hypothesis reassigns the contribution to the corresponding exit channel. It is no longer counted independently as R_{trans}, except through explicitly charged overlap.

If coherent strain alignment is sustained by separated supports, the multi-patch nonlocal strain assignment hypothesis places the contribution either into a coupled coherent support or into a named downstream channel. Thus nonlocal feedback cannot remain outside the channel accounting.

Therefore every active coherent-channel contribution is absorbed, shown to be short-lived relative to stretching time, or reassigned to a named exit channel. Since the coherent-channel margin condition assumes

$$\theta + \delta_{\mathrm{coh}} + \delta_{\mathrm{remaining}} < 1$$

and reserves positive margin for downstream channels, the absorbed coherent-channel cost fits inside the Paper 150J dissipation budget. Hence R^{+}_{patch} and R_{trans} do not remain independent coherent-channel obstructions to the assembly. □

8.5 Combined estimate form

When both coherent channels are directly absorbable pointwise, the theorem yields

$$R^{+}_{\mathrm{patch}}(t) + R_{\mathrm{trans}}(t) \le (\delta_{\mathrm{patch}} + \delta_{\mathrm{trans}})\, D(t) + (C_{\mathrm{patch}} + C_{\mathrm{trans}})\, E_{\omega}(t).$$

Equivalently,

$$R_{\mathrm{coh}}(t) \le \delta_{\mathrm{coh}} D(t) + C_{\mathrm{coh}} E_{\omega}(t).$$

When one or both channels are controlled only in integrated form, the corresponding estimate is

$$\int_{t_0}^{t} R_{\mathrm{coh}}(s)\ \mathrm{d}s \le \delta_{\mathrm{coh}} \int_{t_0}^{t} D(s)\ \mathrm{d}s + C_{\mathrm{coh}} \int_{t_0}^{t} E_{\omega}(s)\ \mathrm{d}s + C^{0}_{\mathrm{coh}}$$

for every

$$t \in I.$$

This is the form compatible with the integrated Gronwall and hidden-spike exclusion lemmas of Paper 150K.

8.6 Role of exit channels

The theorem does not claim that exit channels are controlled. If a coherent route exits to

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad \text{or} \quad R_{\mathrm{path}},$$

then the remaining burden shifts to the paper responsible for that channel.

This is still progress. It means the coherent-channel obstruction has been removed or reclassified. The final assembly can proceed only after the exit channel is controlled, reduced, or absorbed in its own bridge estimate.

Thus the theorem should be read as:

$$\text{coherent channels} \quad \Longrightarrow \quad \text{absorption, finite lifetime, or named dependency.}$$

8.7 What the theorem proves

The theorem proves a conditional dependency result. Under the stated alternatives, R^+_{patch} and R_{trans} cannot remain undefined coherent-channel obstructions. They are either absorbed directly, shown not to persist long enough to remain dangerous, or moved into a named channel dependency.

This is the coherent-channel analogue of the visibility logic in Paper 150L. Paper 150L said danger must become visible. Paper 150M says the first two ordinary visible channels must either be absorbed, have finite lifetime, or become named downstream channels.

8.8 What the theorem does not prove

The theorem does not prove that the aligned-patch alternative or transition-layer alternative holds unconditionally. It does not prove that every aligned patch pays directional-gradient cost, loses alignment, leaks, or has finite lifetime. It does not prove that every transition layer pays enough interface cost, leaks robustly, or has finite lifetime.

It also does not prove that the exit channels

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}$$

are controlled. It only identifies them as downstream dependencies.

Finally, the theorem does not prove margin sharpness. The coefficient condition

$$\theta + \delta_{\mathrm{coh}} + \delta_{\mathrm{remaining}} < 1$$

and the reserve condition

$$\delta_{\mathrm{coh}} \leq (1-\theta) - \delta_{\mathrm{reserve}}$$

remain quantitative bridge requirements.

8.9 Pressure and nonlocal strain

The theorem does not assume that strain alignment is local. Coherent patches and transition layers may be sustained by nonlocal pressure-mediated strain. If such support is absorbed, the estimate must include its effect. If it is not absorbed, it must exit to a named channel.

If separated patches sustain one another through nonlocal strain feedback, the support should be treated as a coupled multi-patch structure or assigned to a downstream channel. Thus pressure and nonlocal strain remain part of the analytic burden. They are not hidden in the channel notation.

8.10 Summary

This section stated the conditional coherent-channel control theorem. Under the aligned-patch and transition-layer alternatives, and assuming no double-counting, multi-patch nonlocal strain assignment, and margin compatibility,

$$R^{+}_{\text{patch}} + R_{\text{trans}}$$

is either absorbable, short-lived relative to stretching time, or reassigned to named downstream channels.

The theorem does not close Navier–Stokes regularity. It narrows the coherent-channel bridge problem: coherent support must either pay cost, lose persistence or alignment, leak robustly, fragment, become scale-local, enter the complement, or become residual pathology. The next section states falsifiers and failure modes for these coherent-channel control targets.

9 Falsifiability and Failure Modes

The previous section stated the conditional coherent-channel control theorem for

$$R^{+}_{\text{patch}} \qquad \text{and} \qquad R_{\text{trans}}.$$

This section identifies how that theorem can fail. The goal is to keep Paper 150M falsifiable. A coherent-channel control paper is useful only if its failure modes are explicit.

The central target is:

$$R^{+}_{\text{patch}} + R_{\text{trans}} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad \text{finite lifetime} \quad \text{or} \quad \text{named exit channel.}$$

A falsifier is therefore a smooth solution, or a controlled limiting sequence of smooth solutions, in which coherent aligned-patch or transition-layer stretching remains significant while avoiding absorbability, finite-lifetime control, and all named exits:

$$R_{\text{frag}}, \quad R_{\text{scale}}, \quad R_{\text{low}}, \quad R_{\text{path}}.$$

9.1 Failure by persistent aligned patch with low directional cost

The most direct aligned-patch failure is a coherent support that preserves positive stretching while paying too little directional-gradient cost.

A schematic failure has

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+ \, \mathrm{d}V$$

significant, while

$$\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |\nabla n|^2 \, \mathrm{d}V$$

remains too small to absorb the patch contribution.

Such a route would show that coherent vorticity direction can preserve dangerous stretching without triggering the expected pinching cost. If the patch also avoids boundary cost, leakage, fragmentation, scale-local transfer, complement assignment, and residual pathology, then

$$R^+_{\mathrm{patch}}$$

remains an independent obstruction.

This is the strongest aligned-patch falsifier.

9.2 Failure by persistence across many stretching times

A sharper aligned-patch falsifier occurs when the patch remains coherent for many characteristic stretching times. Let

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty(\Omega_{\mathrm{patch}}(t))} + \varepsilon}$$

or

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty(\Omega_{\mathrm{patch}}(t))} + \varepsilon}$$

be a local stretching time. If the patch lifetime satisfies

$$\tau_{\mathrm{patch}} \gg \tau_S$$

while positive stretching remains significant and directional-gradient cost remains small, then the route has enough time to produce meaningful enstrophy growth.

This failure is stronger than mere instantaneous coherence. It says the patch is not only visible and aligned, but persistent on the natural amplification timescale. A theorem-level aligned-patch estimate must either bound this lifetime, force a cost, or reclassify the route into a named exit channel.

9.3 Failure by persistent aligned patch with low boundary cost

A second aligned-patch failure occurs if the patch preserves positive stretching while avoiding both directional-gradient cost and boundary or magnitude-gradient cost.

A schematic boundary cost is

$$\mathcal{B}_{\mathrm{patch}}(t) = \nu \int_{\mathcal{N}_r(\partial\Omega_{\mathrm{patch}}(t))} |\nabla\omega|^2 \, \mathrm{d}V.$$

A falsifier has

$$R^+_{\mathrm{patch}}(t)$$

significant, while

$$\mathcal{B}_{\mathrm{patch}}(t)$$

is too small to absorb the contribution, and the route does not leak into

$$R_{\mathrm{trans}}.$$

This would mean that a coherent support can remain both directionally stable and cheaply separated from its surroundings. Such a route would challenge the patch-to-transition logic.

9.4 Failure by broad aligned support without cost

A broad aligned support is dangerous when weak positive alignment over a large region produces significant integrated positive stretching:

$$\int_I \int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+ \, \mathrm{d}V \, \mathrm{d}t.$$

A broad-support falsifier occurs if this integrated contribution is significant, while pointwise alignment remains modest, directional-gradient cost remains small, boundary cost remains small, and the route avoids fragmentation, scale-local transfer, complement assignment, and residual pathology.

This failure mode matters because a proof that only controls compact intense patches may miss wide-area weak alignment. Paper 150M treats broad coherent support as part of

$$R^+_{\mathrm{patch}}.$$

Therefore, a theorem-level aligned-patch estimate must either control broad patches or force them into a named exit channel.

9.5 Failure by coherent transport corridor without cost

A coherent patch or transition-layer structure may behave as a classical transport corridor: a connected or semi-connected region in which vorticity direction, strain alignment, and positive stretching remain organized over time. This terminology is purely fluid-mechanical and does not introduce any non-Navier–Stokes mechanism.

A corridor failure occurs if such an organized region guides positive stretching while paying too little directional-gradient, boundary, or transition-layer cost. In schematic terms, the corridor preserves

$$|\omega|^2 a_+$$

along an organized support while avoiding

$$|\nabla|\omega||^2, \qquad |\omega|^2 |\nabla n|^2,$$

and while avoiding leakage, fragmentation, scale-local transfer, complement assignment, and residual pathology.

This failure would show that the coherent-channel framework needs a sharper transport-corridor estimate: organized transport must either pay cost, lose alignment, leak, fragment, become scale-local, or enter a named residual class.

9.6 Failure by protected core with low transition-layer cost

The strongest transition-layer failure is a protected core that preserves positive stretching while the transition layer pays too little cost.

A schematic failure has

$$P^+_{\mathrm{core}}(t) = \int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+ \, \mathrm{d}V$$

significant, while

$$\mathcal{C}_{\text{trans}}(t) = \nu \int_{\Omega_{\text{trans}}(t)} |\nabla\omega|^2 \ \mathrm{d}V$$

remains too small to absorb it.

If the core remains protected, leakage remains small, alignment remains positive, and the route avoids fragmentation, scale-local transfer, complement support, and residual pathology, then

$$R_{\text{trans}}$$

remains an independent obstruction.

This is the strongest transition-layer falsifier.

9.7 Failure by long-lived protected core

A transition-layer structure may be dangerous because it persists. Even if the instantaneous contribution is not extreme, a long-lived protected core can accumulate significant integrated positive stretching:

$$\int_I P^+_{\text{core}}(t) \ \mathrm{d}t.$$

A long-lived-core falsifier occurs if this integral is significant while the transition-layer cost, leakage, alignment loss, and exit-channel activation remain too small. In that case, the core lifetime is not controlled.

The strongest version occurs when the protected core survives across many characteristic stretching times:

$$\tau_{\text{core}} \gg \tau_S,$$

while remaining positively aligned and insufficiently dissipative. This failure would show that transition-layer protection can delay depletion long enough to matter for enstrophy growth.

9.8 Failure by low leakage

Leakage is one expected route out of transition-layer protection. Recall

$$\mathcal{L}_{\text{leak}}(t) = \frac{\int_{\Omega_{\text{trans}}(t)} |\omega|^2 a_+ \ \mathrm{d}V}{\int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+ \ \mathrm{d}V + \varepsilon}.$$

A leakage failure occurs if the core remains stretching-active while

$$\mathcal{L}_{\text{leak}}(t)$$

stays small and the transition layer pays insufficient gradient cost.

This would mean the core remains isolated without paying enough boundary or transition-layer cost. If the route also avoids fragmentation, scale-local transfer, threshold/complement assignment, and residual pathology, then transition-layer control fails.

9.9 Failure by mask-sensitive leakage

A leakage diagnostic can fail if it depends too strongly on the chosen core-layer mask. Suppose that a sharp mask gives increasing leakage, but a nearby smooth perturbation of the core and transition weights gives a different conclusion. Then the leakage claim is not robust.

In weighted form, the relevant diagnostic is

$$\mathcal{L}_{\mathrm{leak}}{}_{\chi}^{(\rho)}(t) = \frac{\int_\Omega \chi_{\mathrm{trans}}^{(\rho)}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V}{\int_\Omega \chi_{\mathrm{core}}^{(\rho)}(x,t)|\omega|^2 a_+(x,t)\ \mathrm{d}V + \varepsilon}.$$

A mask-sensitive leakage falsifier occurs if the qualitative conclusion changes under admissible smooth variations of

$$\rho.$$

This failure would mean that the transition-layer control is a diagnostic artifact rather than a robust fluid-mechanical estimate. The remedy is to formulate leakage, transition-layer cost, or exit-channel activation using smooth weights, coarse-grained transition regions, or a mask-stable partition family.

9.10 Failure by nonlocal strain support

The strain tensor

$$S_{ij}$$

is nonlocal. A coherent patch or protected core may be sustained by pressure-mediated strain organization rather than by local vorticity geometry alone.

A nonlocal-strain falsifier occurs if nonlocal strain preserves positive stretching on an aligned patch or protected core while all local cost diagnostics remain too small:

$$|\nabla|\omega||^2, \qquad |\omega|^2|\nabla n|^2, \qquad \mathcal{B}_{\mathrm{patch}}, \qquad \mathcal{C}_{\mathrm{trans}}.$$

If the route also avoids leakage, fragmentation, scale-local transfer, complement assignment, and residual pathology, then the coherent-channel framework is missing a nonlocal control mechanism.

This would not invalidate the accounting layer. It would identify nonlocal strain as the analytic obstruction.

9.11 Failure by nonlocal feedback loop

A sharper nonlocal failure mode is a feedback loop between separated coherent structures. Two or more distant patches may contribute to the strain field experienced by each other. In schematic form, one may have supports

$$\Omega_{\mathrm{patch}}^{(1)}(t), \qquad \Omega_{\mathrm{patch}}^{(2)}(t),$$

where the strain alignment on each support is sustained partly by vorticity outside that support.

A nonlocal feedback-loop falsifier occurs if separated patches preserve positive stretching for each other while each local patch pays too little local directional-gradient, boundary, or transition-layer cost. In such a case, neither patch alone carries the full cost of the alignment it receives.

This does not invalidate the channel taxonomy, but it identifies a missing estimate. A theorem-level coherent-channel control result must either:

- assign the nonlocal feedback contribution to the aligned-patch or transition-layer budget;
- show that the coupled patches pay a joint dissipation or interface cost;
- show that the coupled structure fragments, becomes scale-local, or enters the complement;
- or assign the remaining route to residual pathology.

Thus nonlocal feedback is not a loophole, but it may require a multi-patch or nonlocal-strain estimate rather than a purely local patch estimate.

9.12 Failure by moving coherent support

A coherent patch or transition-layer structure may move rapidly through the domain. Fixed spatial diagnostics may fail even if the structure remains coherent in a material frame.

A moving-support falsifier occurs if

$$\int_I \int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 a_+ \, \mathrm{d}V \, \mathrm{d}t$$

or

$$\int_I \int_{\Omega_{\mathrm{core}}(t)} |\omega|^2 a_+ \, \mathrm{d}V \, \mathrm{d}t$$

is significant, while material or stopping-time tracking fails to produce a subinterval-stable estimate.

This would mean the channel is visible in principle but not measurable or trackable in a way compatible with the Paper 150K accounting layer.

The remedy would be a material-frame tracking lemma, a weighted support formulation, or a stopping-time decomposition with summable constants.

9.13 Failure by threshold flicker

A coherent patch or transition-layer structure may flicker across a high-vorticity threshold:

$$\Omega_\kappa(t) = \{x \in \Omega : |\omega(x,t)| > \kappa(t)\}.$$

A threshold-flicker falsifier occurs if the cumulative positive stretching remains significant while the route avoids assignment to

$$R_{\mathrm{low}}, \quad R_{\mathrm{scale}}, \quad \text{or} \quad R_{\mathrm{path}}.$$

This failure would indicate that the threshold/complement machinery is not stable enough. The remedy is to use smooth weights, transition-band diagnostics, or stopping-time visibility across threshold crossings.

9.14 Failure by hidden integrated spike

Paper 150M allows subinterval-stable integrated estimates because coherent patches and transition layers may be moving or intermittent. However, integrated control can fail if it hides a pointwise spike.

A hidden-spike falsifier occurs if an estimate holds only on the full interval

$$[t_0, t_1]$$

but fails to control intermediate times:

$$[t_0, t] \subset I.$$

In that case, a transient coherent patch or protected core may produce a large enstrophy spike before the full-interval average appears controlled.

This is why all integrated estimates in Paper 150M must be subinterval-stable or based on stopping-time partial sums compatible with Paper 150K.

9.15 Failure by nonsummable coherent bursts

A coherent channel may appear through a sequence of burst intervals:

$$I_j = [\tau_j, \tau_{j+1}].$$

A burst failure occurs if each burst appears individually controlled, but the residual constants are not summable:

$$\sum_j C_{0,j} = \infty.$$

Then the burst sequence cannot be inserted into the integrated Gronwall framework.

This failure is especially relevant for moving patches, intermittently protected cores, and threshold-flickering transition layers. The remedy is to prove a finite-budget mechanism, such as dissipation, interface cost, leakage cost, or alignment-loss cost, that forces summability.

9.16 Failure by margin exhaustion

Even if aligned-patch and transition-layer estimates are true, they may consume too much dissipation. A margin-exhaustion failure occurs when

$$R^+_{\mathrm{patch}}(t) \leq \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t),$$

and

$$R_{\mathrm{trans}}(t) \leq \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_\omega(t),$$

but

$$\theta + \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}} + \delta_{\mathrm{remaining}} \geq 1.$$

In that case, the coherent channels are visible and estimated, but not sharply enough for closure. This is a quantitative failure, not a classification failure.

A high-Reynolds-number version of this problem occurs if the effective coefficients approach the full dissipation budget as

$$\nu \to 0.$$

9.17 Failure by coherent-channel budget dominance

A stronger form of margin exhaustion occurs if the coherent channels consume most of the available dissipation budget. Even if

$$\theta + \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}} < 1,$$

the remaining channels may still become impossible to control if

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}$$

is too large.

This is coherent-channel budget dominance. It occurs when the coherent estimates leave too little reserve for

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

In that case, the coherent-channel paper may succeed locally while the full Paper 150J assembly fails globally.

The remedy is sharp coefficient recovery: aligned-patch and transition-layer control must be proved with enough slack to leave a declared positive reserve for downstream channels.

9.18 Failure by double-counting during channel transitions

A patch may become a transition layer, and a transition layer may become fragmented or scale-local. During such transitions, the same positive-stretching contribution must not be counted twice.

A double-counting failure occurs if a contribution is charged simultaneously to

$$R^{+}_{\mathrm{patch}}$$

and

$$R_{\mathrm{trans}},$$

or to

$$R_{\mathrm{trans}}$$

and an exit channel, without an explicit partition or overlap budget.

Paper 150K supplies the accounting rule: exact partitions count once, while bounded overlaps must be charged explicitly. Paper 150M relies on that rule during channel transitions.

9.19 Failure by exit-channel overload

The coherent-channel theorem allows exits to

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

This is useful only if the exit channels are later controlled.

An exit-channel overload failure occurs if aligned patches and transition layers repeatedly exit into another channel in a way that overwhelms the downstream control problem. For example, many patch failures may become fragmentation events, or many transition-layer failures may become scale-local events.

This does not refute Paper 150M by itself. It identifies a dependency for later papers. However, if the exit channel cannot be controlled with margin preserved, then the final assembly still fails.

9.20 What would strengthen Paper 150M

The coherent-channel control program would be strengthened by:

- a theorem showing that persistent aligned patches must pay directional-gradient, boundary, or alignment-loss cost;
- a lifetime estimate comparing patch persistence to the characteristic stretching time;
- a broad-support estimate controlling wide-area weak alignment;
- a transition-layer lifetime estimate;
- a mask-robust leakage lower bound for protected cores;
- a material-frame tracking lemma for moving coherent support;
- a stopping-time summability theorem for coherent bursts;
- a nonlocal-strain estimate linking pressure-mediated alignment to cost or exit channels;
- a multi-patch estimate for nonlocal feedback loops;
- sharp coefficient recovery for δ_{patch} and δ_{trans}.

These would move the coherent-channel program from conditional alternatives toward theorem-level control.

9.21 Summary

Paper 150M can fail in named ways. The strongest aligned-patch falsifier is a persistent coherent patch that preserves positive stretching while paying too little directional-gradient or boundary cost and avoiding all exit channels. The strongest transition-layer falsifier is a protected core that preserves positive stretching while paying too little transition-layer cost, leaking too little, and avoiding all exit channels. The corridor version of the same failure is an organized transport region that preserves positive stretching without paying cost or exiting to a named channel.

Other failures include persistence across many stretching times, mask-sensitive leakage, nonlocal strain support, nonlocal feedback loops, moving-support invisibility, threshold flicker, hidden integrated spikes, nonsummable coherent bursts, margin exhaustion, coherent-channel budget dominance, double-counting during channel transitions, and downstream exit-channel overload. These failure modes identify the exact analytic burdens that later bridge work must address.

10 Relation to Papers 150J, 150K, and 150L

The previous section stated falsifiers and failure modes for coherent aligned-patch and transition-layer control. This section explains how Paper 150M fits into the 150-series architecture. The role of Paper 150M is specific: it begins the ordinary-channel control layer by studying the two most coherent ordinary channels,

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

Paper 150L supplied visibility. Paper 150K supplied accounting. Paper 150J supplied conditional assembly. Paper 150M begins the next step: control of visible coherent channels.

10.1 Relation to Paper 150L: from visibility to control

Paper 150L formulated the universal-entry bridge:

$$\mathcal{D}_{\mathrm{amp}}(I) \Longrightarrow \mathcal{E}_{\mathrm{entry}}(I) \vee \mathcal{C}_{\mathrm{chan}}(I).$$

In expanded form,

$$\mathcal{D}_{\mathrm{amp}}(I) \Longrightarrow \mathcal{E}_{\mathrm{entry}}(I) \vee R^+_{\mathrm{patch}}(I) \vee R_{\mathrm{trans}}(I) \vee R_{\mathrm{frag}}(I) \vee R_{\mathrm{scale}}(I) \vee R_{\mathrm{low}}(I) \vee R_{\mathrm{path}}(I).$$

This was a visibility theorem target. It did not prove that any activated channel is controlled. It only required dangerous amplification to become visible as primary entry or as a named channel.

Paper 150M starts where Paper 150L stops. It assumes that the coherent ordinary channels

$$R^+_{\mathrm{patch}} \quad \text{or} \quad R_{\mathrm{trans}}$$

have become visible, and then asks what must happen next.

The desired next step is:

$$R^+_{\mathrm{patch}} + R_{\mathrm{trans}} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad \text{finite lifetime} \quad \text{or} \quad \text{named exit channel.}$$

Thus Paper 150L answers:

Where is the dangerous stretching?

Paper 150M asks:

Can the first coherent visible routes be controlled or reclassified?

10.2 Relation to Paper 150K: accounting after channel assignment

Paper 150K proved the unconditional accounting lemmas needed once channel assignments and estimates are available. Those lemmas include:

partition-of-unity splitting,

bounded-overlap budget inflation,

$$\text{pointwise Gronwall closure,}$$

$$\text{integrated Gronwall closure,}$$

$$\text{burst summability,}$$

$$\text{superlevel and threshold bookkeeping,}$$

and

$$\text{pathological-reduction bookkeeping.}$$

Paper 150M relies on those accounting lemmas in several ways.

First, when a coherent contribution is assigned to

$$R^+_{\mathrm{patch}} \quad \text{or} \quad R_{\mathrm{trans}},$$

the assignment must be measurable or represented by weights, stopping-time intervals, or a comparable bookkeeping device.

Second, when a contribution exits from

$$R^+_{\mathrm{patch}}$$

to

$$R_{\mathrm{trans}},$$

or from

$$R_{\mathrm{trans}}$$

to

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad \text{or} \quad R_{\mathrm{path}},$$

the same positive stretching must not be counted twice. Paper 150K supplies the no-double-counting rule: exact partitions count once, and bounded overlaps must be charged explicitly.

Third, when a patch or transition layer is controlled only in integrated form, Paper 150K supplies the hidden-spike requirement. The estimate must hold on every subinterval

$$[t_0, t] \subset I$$

or through stopping-time partial sums that control every intermediate time.

Fourth, when coherent channels appear through bursts, Paper 150K supplies the summability requirement:

$$\sum_j C_{0,j} < \infty.$$

Without summability, burst-level control cannot be inserted safely into the final Gronwall closure.

Thus Paper 150M supplies channel-control targets, while Paper 150K supplies the accounting rules that make those targets usable.

10.3 Relation to Paper 150J: contribution to the final assembly

Paper 150J assembled the conditional enstrophy-closure theorem. Its schematic structure was:

$$\text{visibility}+\text{channel control}+\text{pathological closure}+\text{margin preservation}+\text{continuation} \Longrightarrow \sup_{t\in I} E_\omega(t) < \infty.$$

Paper 150M contributes to the channel-control part of this assembly. Specifically, it studies whether the coherent-channel contribution

$$R_{\text{coh}}(t) = R^{+}_{\text{patch}}(t) + R_{\text{trans}}(t)$$

can be controlled by

$$R_{\text{coh}}(t) \le \delta_{\text{coh}} D(t) + C_{\text{coh}} E_\omega(t),$$

where

$$\delta_{\text{coh}} = \delta_{\text{patch}} + \delta_{\text{trans}}.$$

If such an estimate holds, then the Paper 150J margin becomes

$$\theta + \delta_{\text{coh}} + \delta_{\text{remaining}} < 1,$$

where

$$\delta_{\text{remaining}} = \delta_{\text{frag}} + \delta_{\text{scale}} + \delta_{\text{low}} + \delta_{\text{path}}.$$

Paper 150M therefore does not close the Paper 150J theorem by itself. It reduces one part of the margin problem. The remaining ordinary channels and pathological channel must still be controlled, reduced, or absorbed.

10.4 Why aligned patches and transition layers come first

The aligned-patch and transition-layer channels are treated first because they are the most coherent ordinary escape routes.

The aligned-patch channel

$$R^{+}_{\text{patch}}$$

represents a support where vorticity direction remains organized and favorably aligned with strain. This can preserve positive stretching while delaying directional-gradient cost.

The transition-layer channel

$$R_{\text{trans}}$$

represents a protected core and surrounding layer. This can preserve positive stretching while delaying leakage, deformation, or loss of alignment.

These are natural first targets because they represent efficient ways for dangerous stretching to remain coherent. If these channels can be controlled, shown to have finite lifetime, or forced into named exits, then the remaining problem becomes more structured:

$$R_{\text{frag}}, \quad R_{\text{scale}}, \quad R_{\text{low}}, \quad R_{\text{path}}.$$

In this sense, Paper 150M is the first ordinary-channel control paper, not the final ordinary-channel control paper.

10.5 Relation to coherent transport corridors

The coherent-channel structures studied in Paper 150M may also be interpreted as coherent transport corridors in a theory-neutral, classical fluid-mechanical sense. A corridor here means a connected or semi-connected region in which vorticity direction, strain alignment, and positive stretching remain organized over time.

This terminology does not introduce a new force, a new physical postulate, or a non-Navier–Stokes mechanism. It is only a geometric description of organized transport inside the existing velocity and vorticity fields.

In the present paper, a coherent corridor is classified according to its geometry:

$$\text{coherent support} \quad \Longrightarrow \quad R^+_{\text{patch}},$$

while

$$\text{protected core plus mediating layer} \quad \Longrightarrow \quad R_{\text{trans}}.$$

If the corridor fragments, becomes visible only after filtering, crosses a threshold, or avoids ordinary classification, then it exits to

$$R_{\text{frag}}, \quad R_{\text{scale}}, \quad R_{\text{low}}, \quad \text{or} \quad R_{\text{path}}.$$

Thus the corridor language is compatible with Paper 150M only when it remains classical and diagnostic: it describes organized vorticity-strain transport, not an additional mechanism outside Navier–Stokes.

10.6 Relation to fragmentation, scale-local, and complement channels

Paper 150M allows coherent channels to exit into:

$$R_{\text{frag}}, \quad R_{\text{scale}}, \quad R_{\text{low}}.$$

This is not a proof that those exit channels are controlled. It is a dependency map.

If an aligned patch breaks into many stretching-active pieces, the route exits to

$$R_{\text{frag}}.$$

If a patch or transition layer becomes visible only after filtering, the route exits to

$$R_{\text{scale}}.$$

If the support crosses a high-vorticity threshold or lies in the complement of a selected high-vorticity region, the route exits to

$$R_{\text{low}}.$$

These exits identify the next ordinary-channel targets. A later paper must study whether fragmentation, scale-local transfer, and low-vorticity complement stretching are absorbable with margin preserved.

Thus Paper 150M narrows the coherent-channel problem but does not eliminate the downstream ordinary-channel burden.

10.7 Relation to pathological concentration

Paper 150M also allows coherent routes to exit into

$$R_{\mathrm{path}}.$$

This happens when a patch or transition-layer structure preserves significant positive stretching while avoiding absorbability and all ordinary exit channels.

Such a case is not ignored. It becomes residual pathological concentration:

$$R_{\mathrm{path}}.$$

Paper 150I framed the pathological-channel target:

$$R_{\mathrm{path}} \text{ active} \quad \Longrightarrow \quad \text{ordinary-channel reduction or absorbability.}$$

Therefore, a coherent-channel failure into

$$R_{\mathrm{path}}$$

does not close the theorem, but it does name the obstruction. The route is no longer an undefined patch or transition-layer problem. It has become a pathological-concentration problem.

This keeps the dependency chain explicit:

$$R^{+}_{\mathrm{patch}} \text{ or } R_{\mathrm{trans}} \quad \Longrightarrow \quad \text{absorbable cost or named downstream channel.}$$

10.8 Relation to pressure and nonlocal strain

Paper 150L emphasized that the strain tensor

$$S_{ij}$$

is nonlocal. Paper 150M carries that issue into the channel-control layer.

A coherent aligned patch may be sustained by pressure-mediated, nonlocal strain alignment. A protected core may persist because remote flow structure continues to align vorticity with positive strain. These are not separate loopholes. They are part of the analytic burden of controlling

$$R^{+}_{\mathrm{patch}} \quad \text{and} \quad R_{\mathrm{trans}}.$$

A successful aligned-patch or transition-layer estimate must account for nonlocal strain in one of the following ways:

- show that nonlocal strain-supported alignment pays dissipation, boundary, or transition-layer cost;

- show that the support loses positive alignment;
- show that the structure leaks, fragments, becomes scale-local, or enters the complement;
- show that the route becomes residual pathology.

Thus pressure and nonlocal strain remain visible in the proof architecture. They are not hidden inside a local heuristic.

10.9 Relation to multi-patch nonlocal feedback

A sharper nonlocal possibility is multi-patch feedback. Two or more separated coherent supports may sustain each other's strain alignment. In that case, the local cost paid by any single patch may not account for the full positive stretching it receives.

Paper 150M treats this as a coupled-support problem. The relevant support may be

$$\Omega_{\mathrm{patch}}^{\mathrm{coup}}(t) = \bigcup_{m=1}^{M} \Omega_{\mathrm{patch}}^{(m)}(t),$$

with a coupled contribution

$$R_{\mathrm{patch}}^{+}{}^{\mathrm{coup}}(t) = \int_{\Omega_{\mathrm{patch}}^{\mathrm{coup}}(t)} |\omega|^2 a_+(x,t)\ \mathrm{d}V.$$

The theorem target is then not necessarily that each component is individually absorbable. The weaker and more appropriate target is that the coupled structure is absorbable, pays joint cost, fragments, becomes scale-local, enters the complement, or becomes residual pathology.

This prevents nonlocal feedback from becoming an untracked loophole in the coherent-channel program.

10.10 Relation to broad aligned support

Paper 150L identified wide-area low-intensity danger as a possible visibility issue. Paper 150M assigns the coherent version of that issue to the aligned-patch channel.

If weak positive alignment extends over a large coherent support and contributes significantly to

$$\int_I \int_\Omega |\omega|^2 a_+\ \mathrm{d}V\ \mathrm{d}t,$$

then the route belongs to broad aligned-patch control:

$$R_{\mathrm{patch}}^{+}.$$

If the support decomposes into many weakly active pieces, it exits to

$$R_{\mathrm{frag}}$$

or

$$R_{\mathrm{scale}}.$$

If it sits near a threshold or in the complement, it exits to

$$R_{\mathrm{low}}.$$

If no ordinary classification applies, it exits to

$$R_{\mathrm{path}}.$$

This means Paper 150M does not only address compact intense patches. It also recognizes broad coherent alignment as a real coherent-channel target.

10.11 Relation to finite lifetime and stretching times

Paper 150M also adds a time-scale requirement to coherent-channel control. A visible patch or protected core is dangerous only if it persists long enough for positive stretching to accumulate.

The natural comparison is the local stretching time:

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty} + \varepsilon},$$

or the local positive-alignment time:

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty} + \varepsilon}.$$

If a coherent structure lasts for much less than this time, then it may be visible but dynamically lower-order. If it persists across many such times, it becomes a serious obstruction unless it pays cost or exits to another channel.

Thus finite lifetime is itself a possible control route:

$$\text{coherent channel} \quad \Longrightarrow \quad \text{absorbable cost} \quad \text{or} \quad \text{finite lifetime} \quad \text{or} \quad \text{named exit}.$$

10.12 Relation to leakage-mask robustness

For transition layers, Paper 150M also requires leakage diagnostics to be robust. A transition-layer claim should not depend on an arbitrary or jagged choice of core boundary.

For diffuse layers, the core and transition region should be represented by smooth weights

$$\chi^{(\rho)}_{\mathrm{core}}(x,t), \qquad \chi^{(\rho)}_{\mathrm{trans}}(x,t),$$

and the qualitative conclusion should be stable under admissible smooth changes of the boundary-thickness parameter

$$\rho.$$

If a leakage conclusion changes under small smooth mask perturbations, then it is diagnostic rather than theorem-level.

This robustness condition connects Paper 150M directly back to Paper 150K: a channel assignment must be measurable and stable enough to enter the accounting layer.

10.13 Future bridge sequence after Paper 150M

After Paper 150M, the natural next step is to control the remaining ordinary channels. A reasonable sequence is:

Paper 150N: fragmentation, scale-local, and complement control,

Paper 150O: pathological concentration exclusion and reduction,

Paper 150P: margin sharpness and coefficient recovery,

Paper 150Q: bridge assembly after visibility and ordinary-channel control.

Paper 150M should therefore be read as the first ordinary-channel control paper. It handles the coherent channels and passes named failures downstream.

10.14 Summary

Paper 150M connects to Papers 150J, 150K, and 150L as follows:

Paper 150L gives visibility,

Paper 150M begins ordinary-channel control,

Paper 150K gives accounting,

Paper 150J gives conditional assembly.

The specific role of Paper 150M is to address:

$$R^{+}_{\mathrm{patch}} + R_{\mathrm{trans}}.$$

If these coherent channels are absorbable, their coefficients enter the Paper 150J margin. If they are not absorbable, they must have finite lifetime or exit to named downstream channels:

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

Thus Paper 150M narrows the coherent-channel bridge problem: coherent aligned support and protected transition layers must either pay cost, lose persistence or alignment, leak, fragment, become scale-local, enter the complement, or become residual pathology. It also records three additional safeguards: coherent-channel estimates must leave margin reserve, leakage diagnostics must be mask-robust, and nonlocal strain may require coupled multi-patch control.

11 Conclusion

Paper 150M began the ordinary-channel control layer of the 150-series high-vorticity pinching program for the three-dimensional incompressible Navier–Stokes equations. Paper 150L supplied the visibility bridge: dangerous amplification must either enter the primary depletion regime or activate a named channel. Paper 150K supplied the accounting layer needed once channels are visible and estimated. Paper 150J supplied the conditional assembly theorem. The present paper focused on the first two ordinary channels that must be controlled after visibility:

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

The aligned-patch channel represents dangerous positive stretching carried by coherent support where the vorticity direction remains favorably aligned with strain. The transition-layer channel represents dangerous positive stretching carried by a protected core and surrounding layer. These channels are treated first because they are the most coherent ordinary escape routes. They can preserve stretching efficiently while delaying directional-gradient cost, leakage, fragmentation, or loss of alignment.

The paper began from the classical enstrophy balance

$$\frac{\mathrm{d}E_\omega}{\mathrm{d}t} = P(t) - D(t),$$

where

$$P(t) = \int_\Omega \omega_i S_{ij} \omega_j \ \mathrm{d}V$$

is vortex stretching and

$$D(t) = \nu \int_\Omega |\nabla \omega|^2 \ \mathrm{d}V$$

is viscous enstrophy dissipation. The relevant dangerous density is the positive stretching density

$$|\omega|^2 a_+(x,t), \qquad a_+(x,t) = \max\{n_i S_{ij} n_j, 0\}.$$

The geometric dissipation available to oppose such stretching is organized by the decomposition

$$|\nabla \omega|^2 = |\nabla |\omega||^2 + |\omega|^2 |\nabla n|^2.$$

The first term gives magnitude-gradient cost. The second gives directional-gradient cost.

For the aligned-patch channel, the paper defined coherent aligned-patch geometry. A coherent aligned patch is a support

$$\Omega_{\mathrm{patch}}(t)$$

that carries significant positive stretching while the vorticity direction remains coherent and favorably aligned with strain. Such a patch may be compact and intense, broad and weakly aligned, moving, weighted, material, or part of a coupled multi-patch support. Its difficulty is that it may preserve positive stretching while keeping

$$|\nabla n|$$

small.

The aligned-patch control target was stated as

$$R^+_{\mathrm{patch}}(t) \le \delta_{\mathrm{patch}} D(t) + C_{\mathrm{patch}} E_\omega(t),$$

or as the subinterval-stable integrated analogue

$$\int_{t_0}^{t} R^+_{\mathrm{patch}}(s) \ \mathrm{d}s \le \delta_{\mathrm{patch}} \int_{t_0}^{t} D(s) \ \mathrm{d}s + C_{\mathrm{patch}} \int_{t_0}^{t} E_\omega(s) \ \mathrm{d}s + C^0_{\mathrm{patch}}.$$

If direct absorbability fails, the paper identified named exit routes: finite lifetime relative to the characteristic stretching time, alignment loss, leakage into a transition layer, fragmentation, scale-localization, threshold/complement behavior, or residual pathology.

For the transition-layer channel, the paper defined a protected core-layer geometry:

$$\Omega_{\mathrm{core}}(t) \cup \Omega_{\mathrm{trans}}(t).$$

The core carries positive stretching, while the transition layer mediates leakage, deformation, interface cost, alignment loss, or interaction with the surrounding flow. The transition-layer control target was stated as

$$R_{\mathrm{trans}}(t) \leq \delta_{\mathrm{trans}} D(t) + C_{\mathrm{trans}} E_\omega(t),$$

or as the corresponding subinterval-stable integrated estimate. If direct absorbability fails, the route must pay transition-layer cost, leak in a mask-robust way, have finite lifetime, lose core alignment, fragment, become scale-local, enter the threshold/complement channel, or become residual pathology.

The paper then stated the conditional coherent-channel control theorem. Under the aligned-patch alternative, the transition-layer alternative, no double-counting under channel transitions, multi-patch nonlocal strain assignment, and coherent-channel margin compatibility, the coherent channels

$$R^+_{\mathrm{patch}} + R_{\mathrm{trans}}$$

do not remain independent obstructions. They are either absorbed into dissipation and lower-order enstrophy, shown to be short-lived relative to the local stretching time, or reassigned to named downstream channels:

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

In the direct pointwise case, this gives the combined estimate

$$R^+_{\mathrm{patch}}(t) + R_{\mathrm{trans}}(t) \leq \delta_{\mathrm{coh}} D(t) + C_{\mathrm{coh}} E_\omega(t),$$

where

$$\delta_{\mathrm{coh}} = \delta_{\mathrm{patch}} + \delta_{\mathrm{trans}}.$$

This estimate is useful for the Paper 150J assembly only if the final margin remains positive:

$$\theta + \delta_{\mathrm{coh}} + \delta_{\mathrm{remaining}} < 1.$$

Moreover, coherent-channel control should leave a positive reserve for downstream channels rather than merely satisfying the inequality at equality.

The paper made the failure modes explicit. The strongest aligned-patch falsifier is a persistent coherent patch that preserves positive stretching across many stretching times while paying too little directional-gradient or boundary cost and avoiding all exit channels. The strongest transition-layer falsifier is a protected core that preserves positive stretching while the transition layer pays too little cost, leaks too little, and avoids all exit channels. Other failures include broad aligned support without cost, coherent transport corridors without cost, nonlocal strain support, nonlocal feedback loops, moving coherent support, threshold flicker, hidden integrated spikes, nonsummable coherent bursts, margin exhaustion, coherent-channel budget dominance, double-counting during channel transitions, and downstream exit-channel overload.

The pressure and nonlocal strain issue was treated directly. The strain tensor

$$S_{ij}$$

is nonlocal through the velocity field and incompressibility. Therefore, an aligned patch or protected core may be sustained by pressure-mediated strain organization rather than local vorticity geometry alone. Separated patches may also sustain one another through nonlocal strain feedback, requiring a coupled multi-patch estimate rather than a purely local patch estimate. Paper 150M does not hide this difficulty. It treats nonlocal strain support as part of the analytic burden of controlling

$$R^+_{\text{patch}} \quad \text{and} \quad R_{\text{trans}}.$$

A successful estimate must show that such support pays cost, loses alignment, leaks, fragments, becomes scale-local, enters the complement, or becomes residual pathology.

The paper also clarified that coherent aligned patches and transition layers may be viewed as coherent transport corridors in a classical, theory-neutral sense: organized regions in which vorticity direction, strain alignment, and positive stretching remain correlated over time. This language does not introduce a new force or a non-Navier–Stokes mechanism. It is only a geometric description of organized transport within the existing velocity and vorticity fields.

Paper 150M therefore narrows the bridge program. It does not prove full Navier–Stokes regularity. It does not control fragmentation, scale-local transfer, low-vorticity complement stretching, or pathological concentration. It does not recover the final margin by itself. Its contribution is more focused: the two most coherent ordinary channels are converted into explicit control targets, finite-lifetime alternatives, and named failure routes.

In summary, Paper 150M advances the 150-series from visibility to the first ordinary-channel control problem. If coherent aligned patches and transition layers are absorbable, their coefficients enter the Paper 150J margin. If they are not absorbable, they must be short-lived relative to stretching time or exit to named downstream channels. The next step is therefore to control the remaining ordinary channels:

$$R_{\text{frag}}, \quad R_{\text{scale}}, \quad R_{\text{low}},$$

before returning to pathological concentration and margin sharpness in the later bridge papers.

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A Glossary of Terms and Symbols

A.1 Key Terms

Navier–Stokes equations

The classical equations for viscous incompressible fluid motion:

$$\partial_t u + (u \cdot \nabla)u + \nabla p = \nu \Delta u, \qquad \nabla \cdot u = 0.$$

Velocity field The vector field $u(x, t)$ giving the local velocity of the fluid.

Pressure The scalar field $p(x,t)$ enforcing incompressibility. In the vorticity formulation, pressure is not explicit, but its nonlocal effects remain through the velocity gradient and strain tensor.

Incompressibility The condition

$$\nabla \cdot u = 0.$$

It means the fluid does not locally expand or compress.

Vorticity The local rotational part of the flow:

$$\omega = \nabla \times u.$$

Vorticity magnitude

The scalar strength of local rotation:

$$|\omega|.$$

Vorticity direction The unit vector

$$n = \frac{\omega}{|\omega|}$$

where $|\omega| > 0$. It records the direction of local rotation.

Strain tensor The symmetric part of the velocity gradient:

$$S_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i).$$

It measures local stretching and compression.

Vortex stretching The nonlinear three-dimensional amplification mechanism:

$$P(t) = \int_\Omega \omega_i S_{ij} \omega_j \, dV.$$

It is the term that can increase enstrophy.

Enstrophy The total squared vorticity:

$$E_\omega(t) = \frac{1}{2} \int_\Omega |\omega|^2 \, dV.$$

Enstrophy dissipation

The viscous damping term:

$$D(t) = \nu \int_\Omega |\nabla \omega|^2 \, dV.$$

Enstrophy balance The identity

$$\frac{dE_\omega}{dt} = P - D.$$

It says enstrophy grows when vortex stretching exceeds dissipation and decays when dissipation exceeds stretching.

Positive stretching The part of vortex stretching that locally increases enstrophy:

$$|\omega|^2 a_+(x,t), \qquad a_+(x,t) = \max\{n_i S_{ij} n_j, 0\}.$$

Positive stretching density

The nonnegative density

$$|\omega|^2 a_+(x,t).$$

This is the main stretching reservoir assigned to aligned-patch and transition-layer channels.

Positive stretching reservoir

The total positive stretching:

$$P^+(t) = \int_\Omega |\omega|^2 a_+ \, dV.$$

Integrated positive-stretching reservoir

The positive stretching accumulated over a time interval:

$$\mathcal{P}^+(I) = \int_I \int_\Omega |\omega|^2 a_+ \, dV \, dt.$$

This is important for moving, intermittent, broad-support, and transition-layer routes.

High-vorticity pinching program

The 150-series approach that studies whether dangerous high-vorticity amplification must pay geometric cost, lose alignment, leak, fragment, become scale-local, enter the complement, or become residual pathology.

Primary depletion regime

The main high-vorticity pinching/depletion architecture, represented schematically by

$$P(t) \le \theta D(t) + C E_\omega(t) + R_\kappa(t), \qquad 0 \le \theta < 1.$$

Primary depletion coefficient

The coefficient θ in the primary depleted stretching estimate. It measures how much dissipation is consumed before channel remainders are added.

Remainder The part of stretching not controlled by the primary depleted estimate:

$$R_\kappa.$$

Channel decomposition

The splitting of the remainder into named channels:

$$R_\kappa = R^+_{\mathrm{patch}} + R_{\mathrm{trans}} + R_{\mathrm{frag}} + R_{\mathrm{scale}} + R_{\mathrm{low}} + R_{\mathrm{path}}.$$

Ordinary channels The structured non-pathological channels:

$$R^+_{\mathrm{patch}}, \quad R_{\mathrm{trans}}, \quad R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}.$$

Coherent channels The two ordinary channels studied in Paper 150M:

$$R^+_{\mathrm{patch}} \qquad \text{and} \qquad R_{\mathrm{trans}}.$$

Coherent transport corridor

A connected or semi-connected region in which vorticity direction, strain alignment, and positive stretching remain organized over time. In Paper 150M this is only a classical fluid-mechanical description of organized transport inside the Navier–Stokes velocity and vorticity fields. It does not introduce a new force or non-Navier–Stokes mechanism.

Aligned-patch channel

The channel

$$R^+_{\mathrm{patch}}$$

in which dangerous positive stretching is preserved inside coherent support where vorticity direction remains favorably aligned with strain.

Coherent aligned patch

A support $\Omega_{\mathrm{patch}}(t)$ that carries significant positive stretching while vorticity direction remains coherent and favorably aligned with strain.

Patch support The region

$$\Omega_{\mathrm{patch}}(t)$$

assigned to aligned-patch visibility.

Directional coherence

The condition that the vorticity direction n varies slowly on a support. It is measured by directional-gradient cost:

$$\int_{\Omega_{\mathrm{patch}}(t)} |\omega|^2 |\nabla n|^2 \, dV.$$

Directional-gradient cost

The cost associated with changes in vorticity direction:

$$|\omega|^2 |\nabla n|^2.$$

This is central to pinching and aligned-patch control.

Magnitude-gradient cost

The cost associated with changes in vorticity magnitude:

$$|\nabla|\omega||^2.$$

Gradient-cost decomposition

The identity

$$|\nabla\omega|^2 = |\nabla|\omega||^2 + |\omega|^2|\nabla n|^2.$$

It separates magnitude-gradient and directional-gradient contributions to dissipation.

Aligned fraction A diagnostic measuring how much of a patch is dominantly positively aligned with strain:

$$\Pi^+_{\text{patch}}(t) = \frac{\int_{\Omega_{\text{patch}}(t)} |\omega|^2 a_+ \, dV}{\int_{\Omega_{\text{patch}}(t)} |\omega|^2 |a| \, dV + \varepsilon}.$$

Patch persistence The condition that an aligned patch remains significant long enough to contribute meaningfully to integrated positive stretching.

Characteristic stretching time

A local time scale measuring how quickly strain can amplify vorticity:

$$\tau_S(t) \sim \frac{1}{\|S(t)\|_{L^\infty} + \varepsilon}.$$

A related positive-alignment time is

$$\tau_a(t) \sim \frac{1}{\|a_+(t)\|_{L^\infty} + \varepsilon}.$$

Finite lifetime route

A control route in which an aligned patch or protected core does not persist long enough, relative to the characteristic stretching time, to produce dangerous integrated growth.

Broad aligned support

A coherent aligned region that is not a compact intense hotspot. Weak positive alignment over a large region can still matter if the integrated positive-stretching contribution is significant.

Patch boundary cost

A cost associated with the boundary or interface of an aligned patch, often represented schematically by

$$\mathcal{B}_{\text{patch}}(t) = \nu \int_{\mathcal{N}_r(\partial\Omega_{\text{patch}}(t))} |\nabla\omega|^2 \, dV.$$

Transition-layer channel

The channel

$$R_{\text{trans}}$$

in which a stretching-active core is protected, insulated, or delayed by a surrounding boundary or transition region.

Transition-layer structure

A core-layer geometry

$$\Omega_{\text{core}}(t) \cup \Omega_{\text{trans}}(t)$$

where the core carries positive stretching and the transition layer mediates leakage, deformation, gradient cost, or alignment loss.

Core The stretching-active part of a transition-layer structure:

$$\Omega_{\text{core}}(t).$$

Transition region The surrounding layer:

$$\Omega_{\text{trans}}(t).$$

It mediates leakage, deformation, interface cost, directional-gradient cost, magnitude-gradient cost, or alignment loss.

Core positive stretching

The positive stretching carried by the core:

$$P^+_{\text{core}}(t) = \int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+ \, dV.$$

Core fraction The fraction of positive stretching carried by the core:

$$I^+_{\text{core}}(t) = \frac{\int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+ \, dV}{\int_{\Omega_{\text{core}}(t) \cup \Omega_{\text{trans}}(t)} |\omega|^2 a_+ \, dV + \varepsilon}.$$

Transition-layer cost

The gradient cost paid inside a transition region:

$$\mathcal{C}_{\text{trans}}(t) = \nu \int_{\Omega_{\text{trans}}(t)} |\nabla\omega|^2 \, dV.$$

Leakage The movement of vorticity magnitude, directional coherence, or positive strain alignment from a protected core into the surrounding transition layer.

Leakage diagnostic A schematic ratio measuring how much positive stretching has moved into the transition layer:

$$\mathcal{L}_{\text{leak}}(t) = \frac{\int_{\Omega_{\text{trans}}(t)} |\omega|^2 a_+ \, dV}{\int_{\Omega_{\text{core}}(t)} |\omega|^2 a_+ \, dV + \varepsilon}.$$

Mask-robust leakage

Leakage that remains qualitatively stable under smooth changes of the core-layer masks. This prevents a transition-layer conclusion from depending on an arbitrary or jagged boundary choice.

Weighted leakage diagnostic

A leakage diagnostic using smooth core and transition weights:

$$\mathcal{L}^{(\rho)}_{\mathrm{leak},\chi}(t) = \frac{\int_\Omega \chi^{(\rho)}_{\mathrm{trans}} |\omega|^2 a_+ \, dV}{\int_\Omega \chi^{(\rho)}_{\mathrm{core}} |\omega|^2 a_+ \, dV + \varepsilon}.$$

Core lifetime The duration or time set over which a protected core remains stretching-active and carries a significant share of positive stretching.

Alignment loss A route by which a patch or core stops being dangerous because the positive strain-alignment factor a_+ decreases.

Channel transition A reclassification of the same positive-stretching contribution from one channel to another, such as

$$R^+_{\mathrm{patch}} \to R_{\mathrm{trans}}$$

or

$$R_{\mathrm{trans}} \to R_{\mathrm{frag}}.$$

Exit channel A named downstream channel entered when aligned-patch or transition-layer control fails. Possible exits include

$$R_{\mathrm{frag}}, \quad R_{\mathrm{scale}}, \quad R_{\mathrm{low}}, \quad R_{\mathrm{path}}.$$

Fragmentation channel

The channel in which positive stretching is distributed across many separated or semi-separated components:

$$R_{\mathrm{frag}}.$$

Scale-local channel The channel in which positive stretching becomes visible only after filtering or scale decomposition:

$$R_{\mathrm{scale}}.$$

Low-vorticity complement channel

The channel in which positive stretching lies outside the selected high-vorticity region, near a threshold boundary, or in the complement of a smooth high-vorticity weight:

$$R_{\mathrm{low}}.$$

Pathological concentration channel
The residual channel

$$R_{\text{path}}$$

assigned to dangerous concentration after ordinary-channel tests fail.

Residual pathology
A route that preserves significant positive stretching while avoiding absorbability and all ordinary exit channels.

Multi-patch nonlocal feedback
A route in which two or more separated coherent supports sustain each other's strain alignment through the nonlocal strain field. Such a route may require a coupled multi-patch estimate rather than separate local patch estimates.

Coupled multi-patch support
A union of separated coherent supports treated as one nonlocal feedback system:

$$\Omega_{\text{patch}}^{\text{coup}}(t) = \bigcup_{m=1}^{M} \Omega_{\text{patch}}^{(m)}(t).$$

No double-counting
The requirement that the same positive-stretching contribution not be charged simultaneously to multiple channels unless an explicit overlap budget is included.

Subinterval-stable integrated estimate
An estimate that holds on every subinterval

$$[t_0, t] \subset I.$$

This is required to avoid hidden spikes in integrated channel control.

Hidden spike A failure mode in which a full-interval integrated estimate appears controlled while a large intermediate enstrophy spike occurs.

Coherent burst A short interval on which an aligned-patch or transition-layer contribution becomes significant.

Nonsummable coherent bursts
A failure mode in which burst constants satisfy

$$\sum_j C_{0,j} = \infty.$$

Then burst-level control cannot be inserted into the integrated Gronwall framework.

Margin preservation
The requirement that the total dissipation coefficient remain below one:

$$\theta + \delta_{\text{patch}} + \delta_{\text{trans}} + \delta_{\text{remaining}} < 1.$$

Margin exhaustion A failure mode in which estimates exist, but their coefficients consume all available dissipation:

$$\theta + \delta_{\text{patch}} + \delta_{\text{trans}} + \delta_{\text{remaining}} \geq 1.$$

Coherent-channel budget dominance
A failure mode in which the coherent-channel coefficient

$$\delta_{\text{coh}} = \delta_{\text{patch}} + \delta_{\text{trans}}$$

uses so much of the available dissipation budget that insufficient reserve remains for downstream channels.

Dissipation reserve A positive margin reserved for downstream channels:

$$\delta_{\text{reserve}} > 0.$$

Paper 150M requires coherent-channel estimates to leave such reserve for fragmentation, scale-local transfer, complement behavior, and pathological concentration.

Nonlocal strain The fact that S_{ij} depends nonlocally on velocity and vorticity through incompressibility and pressure. Paper 150M treats this as part of the aligned-patch and transition-layer control problem.

Pressure-mediated alignment
Positive strain alignment sustained by nonlocal pressure or velocity-gradient effects rather than purely local vorticity geometry.

Conditional coherent-channel control theorem
The Paper 150M theorem stating that, under the aligned-patch alternative, transition-layer alternative, no-double-counting rule, multi-patch nonlocal strain assignment, and margin compatibility,

$$R^{+}_{\text{patch}} + R_{\text{trans}}$$

is absorbable, short-lived relative to stretching time, or reassigned to named downstream channels.

A.2 Symbols

Table 1: Symbols used in Paper 150M.

Symbol	Pronunciation	Meaning
u	"u"	Velocity field
p	"p"	Pressure field
ν	"nu"	Kinematic viscosity
Ω	"Omega"	Spatial domain
$\mathbb{T}^3$	"three-torus"	Periodic three-dimensional domain
t	"t"	Time
∇	"gradient" or "del"	Spatial derivative
Δ	"Laplacian"	Diffusion operator
ω	"omega"	Vorticity field
$\lvert\omega\rvert$	"omega magnitude"	Vorticity strength
n	"n"	Unit vorticity direction
S_{ij}	"S i j"	Strain tensor
a	"a"	Alignment factor $n_i S_{ij} n_j$
a_+	"a plus"	Positive part of the alignment factor
E_ω	"E omega"	Enstrophy
P	"P"	Vortex stretching
P^+	"P plus"	Positive stretching reservoir
D	"D"	Enstrophy dissipation
R_κ	"R kappa"	Full remainder
R^+_{patch}	"R patch plus"	Aligned-patch channel
R_{trans}	"R transition"	Transition-layer channel
R_{frag}	"R fragmentation"	Fragmentation channel
R_{scale}	"R scale"	Scale-local channel
R_{low}	"R low"	Low-vorticity complement channel
R_{path}	"R path"	Pathological concentration channel
R_{coh}	"R coherent"	Combined coherent-channel contribution $R^+_{\mathrm{patch}} + R_{\mathrm{trans}}$
Ω_κ	"Omega kappa"	High-vorticity region above threshold κ
Ω_{patch}	"Omega patch"	Aligned-patch support
Ω_{core}	"Omega core"	Stretching-active core
Ω_{trans}	"Omega transition"	Transition-layer region
P^+_{patch}	"P patch plus"	Positive stretching carried by an aligned patch
P^+_{core}	"P core plus"	Positive stretching carried by a core
Π^+_{patch}	"Pi patch plus"	Positive aligned fraction on a patch
I^+_{core}	"I core plus"	Fraction of positive stretching carried by the core
$\mathcal{Q}_{\mathrm{patch}}$	"Q patch"	Directional-coherence ratio for a patch
$\mathcal{B}_{\mathrm{patch}}$	"B patch"	Patch boundary or interface cost
$\mathcal{C}_{\mathrm{trans}}$	"C transition"	Transition-layer cost

Continued on next page

Symbol	Pronunciation	Meaning
$\mathcal{L}_{\text{leak}}$	"L leak"	Leakage diagnostic
θ	"theta"	Primary depletion dissipation coefficient
δ_{patch}	"delta patch"	Aligned-patch dissipation coefficient
δ_{trans}	"delta transition"	Transition-layer dissipation coefficient
δ_{coh}	"delta coherent"	Combined coherent-channel coefficient
$\delta_{\text{remaining}}$	"delta remaining"	Dissipation coefficient for downstream channels
δ_{reserve}	"delta reserve"	Positive dissipation reserve for downstream channels
C_{patch}	"C patch"	Lower-order enstrophy coefficient for aligned patches
C_{trans}	"C transition"	Lower-order enstrophy coefficient for transition layers
C_{coh}	"C coherent"	Combined lower-order coefficient for coherent channels
C^0_{patch}	"C patch zero"	Integrated aligned-patch residual constant
C^0_{trans}	"C transition zero"	Integrated transition-layer residual constant
C^0_{coh}	"C coherent zero"	Integrated coherent-channel residual constant
I	"I"	Time interval
I_j	"I j"	Burst or stopping-time interval
τ_j	"tau j"	Stopping-time endpoint
τ_S	"tau S"	Characteristic strain/stretching time
τ_a	"tau a"	Positive-alignment stretching time
τ_{patch}	"tau patch"	Lifetime of an aligned patch
τ_{core}	"tau core"	Lifetime of a protected core
$A_j(t)$	"A j of t"	Component in a fragmented support
N^+_{eff}	"N effective plus"	Effective positive component count
π^+_j	"pi j plus"	Positive-stretching fraction in component j
$\mathcal{L}$	"script L"	Declared scale family
ℓ	"ell"	Filter scale
G_ℓ	"G ell"	Filter at scale ℓ
$\Omega^{\text{coup}}_{\text{patch}}$	"Omega patch coupled"	Coupled multi-patch support
$R^{\text{coup}}_{\text{patch}}$	"R patch coupled"	Coupled aligned-patch contribution
$\chi^{(\rho)}_{\text{core}}$	"chi core rho"	Smooth core weight with boundary-thickness parameter ρ
$\chi^{(\rho)}_{\text{trans}}$	"chi transition rho"	Smooth transition-layer weight with boundary-thickness parameter ρ
$\mathcal{L}^{(\rho)}_{\text{leak},\chi}$	"L leak chi rho"	Weighted mask-robust leakage diagnostic
ρ	"rho"	Smoothing or boundary-thickness parameter for core-layer masks
ε	"epsilon"	Small regularization constant

B Plain-Language Summary

B.1 Plain-Language Summary

This paper is about the first two visible ways that dangerous fluid stretching can hide.

The Navier-Stokes equations describe how fluids move. In three dimensions, spinning parts of a fluid can stretch. When spinning fluid stretches, its spin can become stronger. This can make the equations hard to control.

Earlier papers in the 150-series separated the problem into layers. Paper 150L said dangerous stretching must become visible. Paper 150K proved the bookkeeping rules for counting visible channels. Paper 150J showed how the pieces would combine if the needed estimates are proved.

Paper 150M starts controlling the visible channels. It focuses on two channels: aligned patches and transition layers.

An aligned patch is a region where the spinning direction of the fluid stays organized and lines up with stretching. This can be dangerous because the fluid keeps stretching in a helpful direction without quickly paying directional cost. A transition layer is a protected core with a surrounding boundary region. The core keeps stretching, while the boundary delays leakage, deformation, or loss of alignment.

The paper also describes these structures as coherent transport corridors. This only means organized regions where vorticity direction, strain alignment, and stretching stay connected over time. It does not add a new force. It is just a way to describe organized transport in the ordinary Navier-Stokes equations.

The paper asks what must happen to these channels. Either they must be absorbed by dissipation and lower-order enstrophy, or they must change into another named channel. An aligned patch may leak into a transition layer. It may fragment into many pieces. It may become visible only at a certain scale. It may move into the low-vorticity complement. Or it may become residual pathology.

The paper also adds three safeguards. First, a patch or core is most dangerous if it lasts longer than the local stretching time. Second, leakage must be robust: it should not depend on a fragile or arbitrary boundary choice. Third, coherent-channel control must leave enough dissipation budget for the remaining channels.

The paper does not prove full Navier-Stokes regularity. It does not control every possible channel. It focuses on the two most coherent ordinary channels and turns them into clear control targets.

B.2 More Intuitive Analogy

Imagine a spinning rope in water.

If the rope stays straight and lined up with the pull, it can keep stretching efficiently. That is like an aligned patch.

Paper 150M asks whether the rope must eventually bend, fray, leak energy, break into pieces, or pay some cost.

B.3 Second Intuitive Analogy

Imagine a campfire protected by a ring of stones.

The fire is the active core. The stones are the transition layer. The ring may keep the fire strong for a while by protecting it from wind.

Paper 150M asks whether that protective ring eventually heats up, cracks, leaks, breaks apart, or stops protecting the fire.

B.4 Third Intuitive Analogy

Think of a crowd walking in the same direction.

If everyone stays aligned, the crowd moves efficiently. If people start turning, bumping, spreading out, or breaking into groups, the motion changes.

An aligned patch is like the organized crowd. Fragmentation is when the crowd breaks into groups.

B.5 Fourth Intuitive Analogy: Visibility Is Not Control

Seeing a problem is not the same as fixing it.

Paper 150L helped make dangerous stretching visible. Paper 150M starts asking whether the first visible problems can actually be controlled.

B.6 Fifth Intuitive Analogy: A Protected Core

Imagine a hot coal wrapped in insulation.

The coal can stay hot because the insulation slows heat loss. But the insulation may eventually heat up, crack, leak, or fail.

A transition layer is like that insulation. It may protect stretching for a while, but protection should have a cost.

B.7 Sixth Intuitive Analogy: Named Exits

Imagine a hallway with labeled doors.

If a person cannot stay in the hallway, they must leave through a labeled door: stairs, elevator, side room, or emergency exit.

Paper 150M says aligned patches and transition layers must either be controlled or exit through named channels.

B.8 Seventh Intuitive Analogy: The Budget

Imagine repairing leaks with a limited budget.

Even if each leak can be patched, the repair only works if the total cost stays below the budget.

Paper 150M keeps track of this. The aligned-patch and transition-layer costs must leave enough dissipation budget for the remaining channels.

B.9 Eighth Intuitive Analogy: A Smooth Road

Imagine cars moving along a smooth road.

If the road stays connected and everyone moves in the same direction, traffic flows easily. That is like a coherent transport corridor. The danger is that smooth flow can keep going without paying much cost.

B.10 Ninth Intuitive Analogy: A Stopwatch

A spark is less dangerous if it dies quickly.

But if it lasts long enough to light nearby material, it matters. An aligned patch is similar. Its danger depends not only on how strong it is, but how long it lasts compared with the stretching time.

B.11 Tenth Intuitive Analogy: Drawing the Boundary

Imagine drawing a circle around a spill.

If moving the circle a little changes whether the spill looks contained, the measurement is not reliable. Leakage in this paper must be stable under small, smooth changes of the boundary.

B.12 Thirty-Word Summary

Paper 150M studies coherent stretching corridors: aligned patches and transition layers must pay cost, remain short-lived, leak robustly, fragment, become scale-local, enter complement, or become pathology.